Production Forecasting of Multistage Hydraulically Fractured Horizontal Wells in Shale Gas Reservoirs with Radial Flow

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***Abstract*—Transient linear flow is commonly considered as the dominant flow regime in multistage hydraulically fractured horizontal wells in shale gas reservoirs. A slab transient dual porosity model was built and skin effect was studied to interpret the early period production performance. However, the mechanism of early period performance characteristics is not revealed by skin effect. When flow from the vertical fracture into the horizontal wellbore, gas converges in the near-wellbore-zone due to the decreasing flow area. In other words, besides linear flow, there is another flow behavior in vertical fracture: radial flow. This paper presents a slab transient dual porosity model with radial flow effect in natural fractures. The model is consists of three flow regimes: fracture radial flow, fracture linear flow and matrix linear flow. Analytical solution is obtained by Laplace transformation and type curves are drawn. The results show that different from skin effect model, the radial flow effect not only results in severe reduction of early period production performance, but also the late period production performance.**

***Index Terms*—shale gas, fractured horizontal well, radial flow, production forecasting**

I. Introduction

China is rich in shale gas and efficient development of shale gas is vital for energy security as well as energy structure improvement of China. Shale gas is one kind of unconventional natural gas, which is difficult to develop and demands a large amount of investment. Accurate productivity forecasting offers a key foundation for decisions made in shale gas development. Multistage hydraulically fractured horizontal wells have been widely used in the past several years to develop shale gas industrially and commercially. Analysis of type curves show that transient linear flow is the dominant flow regime in fractured wells in shale gas reservoirs, due to the extremely low permeability of shale matrix.

El-Banbi [1] developed a transient dual-porosity model (slab model) for unconventional reservoirs. Bello and Wattenbarger [2]-[3] discussed the type curves of slab model and four flow regimes are studied: fracture linear flow, bilinear flow, matrix linear flow, boundary-dominated flow. Since fracture linear flow or bilinear flow behaviors are generally cannot explain the early-term shale gas wells performance characteristics, a skin effect for matrix linear flow is described to modify the shape of type curves (Bello and Wattenbarger [4]; Nobakht and Clarkson [5]). The transient linear dual-porosity model has been widely used for production data analysis of fractured shale gas wells (Abdulal *et al.* [6]; Moghadam et al [7]; Xu *et al.* [8]). Many studies focused on developing the transient linear dual-porosity model (Ozkan *et al.* [9]; Brohi *et al.* [10]; Tivayanonda *et al.* [11]; Xu *et al.* [12]).

However, the mechanism of early-term performance characteristics is not revealed by skin effect. A more appropriate interpretation for this phenomenon is that gas converges in the near-wellbore-zone due to the decreasing flow area when flow from the vertical fracture into the horizontal wellbore. In other words, besides linear flow, there is another flow behavior in vertical fracture: radial flow. Compare with linear flow, radial flow results in additional pressure drop in the near-wellbore-zone and reduces the productivity of fractured horizontal wells.

In this paper, based on the Bello's research, a fully transient dual-porosity model is established to describe three flow systems: linear flow from matrix to natural fractures, linear flow from natural fractures to near-wellbore-zone; radial flow from near-wellbore-zone to wellbore. The new analytical solution is developed to calculate type curves. The results show that different from skin effect model, the radial flow effect not only results in severe reduction of early period production performance, but also the late period production performance.

II. Physical Model

Natural fractures are widespread in shale gas reservoirs. After large scale multistage hydraulically fractured, the natural fractures open and form in an interconnected fracture network. The network is the dominant pathway for gas, called Stimulated Reservoir Volume (SRV). The process for gas flowing from the reservoir to wellbore is consists of three parts: matrix to natural fractures, natural

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fractures to hydraulic fractures and from hydraulic fractures to wellbore. Flow conductivity of hydraulic fractures and the wellbore is much higher than matrix and natural fractures and can be treated as infinite conductivity. Assuming SRV is filled with natural fractures, a slab transient dual porosity model is built, as is shown in Fig. 1. Other assumptions are as follows:

- The horizontal well is placed in the center of a horizontal reservoir with uniform thickness;
- Single gas phase flows in matrix and fracture system, taking account of absorption and desorption, diffusion effects of natural gas in matrix nanometer pores;
- Three flow regimes occur: linear flow from matrix to natural fractures, linear flow from natural fractures to near-wellbore-zone; radial flow from near-wellbore-zone to wellbore.

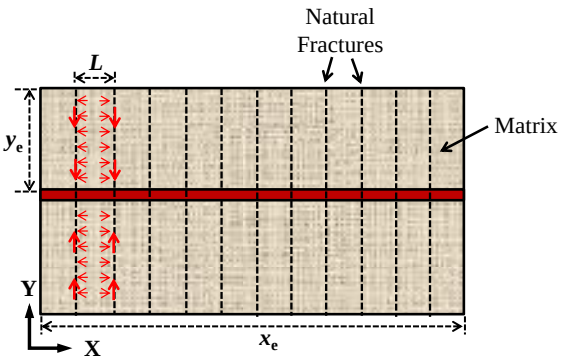

Figure 1. Physical model for multistage hydraulically fractured horizontal wells in shale gas reservoirs.

III. Mathematical Model

A. Matrix Linear Flow

According to the previous assumptions, the direction parallel to horizontal well is set as the x axis. A quarter of the slab matrix is extracted randomly as the research object. As the shale matrix is extremely dense, the governing equation of transient linear flow from matrix to natural fractures is

$$\frac{\partial}{\partial x}(-\rho v_m)=\frac{\partial(\rho\varphi_m)}{\partial t} \tag{1}$$

where φ_m is the matrix porosity, ρ is density of natural gas, t is time.

The flow velocity for gas in nanometer pores matrix is decided by Darcy velocity and diffusion velocity. Introducing apparent permeability k_{ma}:

$$k_{ma}=k_m(1+\frac{\mu D c_g}{k_m}) \tag{2}$$

where μ is gas viscosity, k_m is matrix permeability, c_g is gas compressibility, D is matrix diffusion coefficient.

Than velocity becomes:

$$v_m=-\frac{k_{ma}}{\mu}\frac{\partial p_m}{\partial x} \tag{3}$$

where p_m is matrix pressure.

As well as free gas, there is a large amount of absorbed gas in shale matrix. The effect of absorption and desorption is taken into account by added a desorption compressibility into total compressibility. Introducing matrix pseudo-pressure term ψ_m, the governing equation for matrix linear flow becomes:

$$\frac{\partial^2\psi_m}{\partial x}=\frac{\varphi_m C_{tm}\mu(\bar{p})}{k_{ma}}\frac{\partial\psi_m}{\partial t} \tag{4}$$

where $c_{tm}=c_m+c_g+c_d$, c_m and c_d are matrix compressibility and desorption compressibility, respectively.

The center of matrix slab is considered as closed boundary, then the initial boundary condition and boundary conditions of matrix linear flow respectively are:

$$\begin{cases}\psi_m(x,t)\big|_{t=0}=0\\ \dfrac{\partial\psi_m(x,t)}{\partial x}\Big|_{x=0}=0\\ \psi_m(x,t)\big|_{x=\frac{L}{2}}=\psi_f\end{cases} \tag{5}$$

B. Fracture Linear Flow

The governing equation for transient linear flow from natural fractures to near-wellbore-zone can be stated as:

$$\frac{\partial^2\psi_f}{\partial y^2}=\frac{\varphi_f c_{ft}\mu(\bar{p})}{k_f}\frac{\partial\psi_f}{\partial t}-\frac{2k_{ma}}{Lk_f}\frac{\partial\psi_m}{\partial x}\Big|_{x=\frac{L}{2}} \tag{6}$$

where ψ_f is pseudo-pressure for fracture linear flow, φ_f, c_{ft}, and k_f are porosity, compressibility and permeability of fracture, respectively.

The last term of equation (6) represents the influence of gas flowing from matrix system to natural fracture system. Since the contribution of SRV outside is neglected, the initial boundary condition and outer boundary condition of fracture linear flow respectively are:

$$\begin{cases}\psi_f(y,t)\big|_{t=0}=0\\ \dfrac{\partial\psi_f(y,t)}{\partial y}\Big|_{y=y_e}=0\end{cases} \tag{7}$$

where y_e is fracture length.

C. Fracture Radial Flow

When flow from the vertical fracture into the horizontal wellbore, gas converges in the near-wellbore-zone due to the decreasing flow area. Compare with linear flow, radial flow results in additional pressure drop in the near-wellbore-zone and reduces the productivity of fractured horizontal wells. The transient governing equation for fracture radial flow is:

$$\frac{\partial^2\psi_r}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_r}{\partial r}=\frac{\varphi_f c_{ft}\mu(\bar{p})}{k_f}\frac{\partial\psi_r}{\partial t} \tag{8}$$

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where ψ_r is pseudo-pressure for fracture radial flow.

TABLE I. DEFINITION OF DIMENSIONLESS VARIABLES

Dimensionless Variables	Equation
Dimensionless Pseudo-pressure	$\psi_D = \dfrac{k_f w_f (\psi_i - \psi)}{1.291\times10^{-3} qT}$
Dimensionless Pseudo-time	$t_{Da} = \dfrac{k_f t_a}{\mu(\varphi_m c_{tm} + \varphi_f c_{tf}) A_{cw}}$
Dimensionless Length	$x_D = \dfrac{x}{L/2}$ $\quad y_D = \dfrac{y}{\sqrt{A_{cw}}}$ $\quad y_{eD} = \dfrac{y_e}{\sqrt{A_{cw}}}$ $\quad r_D = \dfrac{r}{\sqrt{A_{cw}}}$
Dimensionless Interporosity coefficient	$\lambda_{cw} = \dfrac{12}{L^2}\dfrac{k_{ma}}{k_f} A_{cw}$
Dimensionless storativity ratio	$\omega = \dfrac{\varphi_f c_{tf}}{\varphi_m c_{tm} + \varphi_f c_{tf}}$

At the interface of fracture linear flow and fracture radial flow ($r=r_c$), both the mass velocity and pseudo-pressure are equal, then the continuity conditions can be obtained:

$$\begin{cases} \left.\dfrac{\partial \psi_r(r,t)}{\partial r}\right|_{r=r_c} = \dfrac{2}{\pi}\left.\dfrac{\partial \psi_f(r,t)}{\partial y}\right|_{y=r_c} \\ \psi_r(r,t)|_{y=r_c} = \psi_f(r,t)\big|_{r=r_c} \end{cases} \tag{9}$$

As flow resistance of hydraulic fractures and wellbore is neglected, assuming hydraulically fractured horizontal wells producing with constant rate, the inner boundary condition is:

$$r\left.\frac{\partial \psi_r}{\partial r}\right|_{r=r_w} = -\frac{1.291\times10^{-3} qT}{k_f w_f} \tag{10}$$

where T is reservoir temperature, w_f is fracture width, r_w is the wellbore radius.

where $A_{cw}=2x_e h$, h is reservoir thickness and x_e is horizontal well length.

IV. MODEL SOLUTION

In order to solve the model, dimensionless variables are introduced. The definition of these variables is given in Table I. Laplace Transformed also used and the governing equation of matrix linear flow in Laplace space is:

$$\frac{d^2\bar{\psi}_{mD}}{dx_D^2} = \frac{3}{\lambda}(1-\omega)s\bar{\psi}_{mD} \tag{11}$$

The General solution for equation (10) is obtained and substituted into the boundary condition, then

$$\bar{\psi}_{mD} = \frac{\bar{\psi}_{fD}\cosh[\sqrt{\dfrac{3(1-\omega)s}{\lambda_{cw}}}\cdot x_D]}{\cosh\sqrt{\dfrac{3(1-\omega)s}{\lambda_{cw}}}} \tag{12}$$

The governing equation for fracture linear flow in Laplace space is:

$$\frac{\partial^2\bar{\psi}_{fD}}{\partial y_D^2} = \omega s\bar{\psi}_{fD} - \frac{\lambda_{cw}}{3}\left.\frac{\partial\bar{\psi}_{mD}}{\partial x_D}\right|_{x_D=1} \tag{13}$$

Substituting equation (11) is into equation (12), the general solution of equation (12) is:

$$\bar{\psi}_{fD} = A_1\cosh[\sqrt{sf(s)}\cdot y_D] + B_1\sinh[\sqrt{sf(s)}\cdot y_D] \tag{14}$$

In which:

$$f(s) = \omega + \frac{\lambda_{cw}}{3}\sqrt{\frac{3(1-\omega)}{\lambda_{cw}s}}\tanh\sqrt{\frac{3(1-\omega)s}{\lambda_{cw}}} \tag{15}$$

The general solution for fracture radial flow in Laplace space is:

$$\bar{\psi}_{rD} = A_2 I_0(r_D\sqrt{\omega s}) + B_2 K_0(r_D\sqrt{\omega s}) \tag{16}$$

According to the outer boundary condition of fracture linear flow, the following equation is obtained:

$$B_1 = -A_1\tanh[\sqrt{sf(s)}\cdot y_{eD}] \tag{17}$$

Inner boundary condition of fracture linear flow can be stated as:

$$I_1(r_{wD}\sqrt{\omega s})\sqrt{\omega s}\cdot A_2 - K_1(r_{wD}\sqrt{\omega s})\sqrt{\omega s}\cdot B_2 = -\frac{1}{sr_{wD}} \tag{18}$$

The continuity conditions can be changed into:

$$\begin{cases} I_1(r_{cD}\sqrt{\omega s})\sqrt{\omega s}\cdot A_2 - K_1(r_{cD}\sqrt{\omega s})\sqrt{\omega s}\cdot B_2 \\ = \dfrac{2}{\pi}[A_1\sinh(\sqrt{sf(s)}\cdot r_{cD})\sqrt{sf(s)} + B_1\cosh(\sqrt{sf(s)}\cdot r_{cD})\sqrt{sf(s)}] \\ I_0(r_{cD}\sqrt{\omega s})\cdot A_2 + K_0(r_{cD}\sqrt{\omega s})\cdot B_2 \\ = A_1\cosh[\sqrt{sf(s)}\cdot r_{cD}] + B_1\sinh[\sqrt{sf(s)}\cdot r_{cD}] \end{cases} \tag{19}$$

where A_1, B_1, A_2, B_2 are coefficients for governing equation's general solutions and can be obtained by Equation (17)~(19).

Than the pseudo-pressure at the wellbore is:

$$\bar{\psi}_{rD} = A_2 I_0(r_{wD}\sqrt{\omega s}) + B_2 K_0(r_{wD}\sqrt{\omega s}) \tag{20}$$

After all, the solution for dimensionless productivity in Laplace space becomes as follows and dimensionless productivity in real space can be obtained by Stefest inversion method:

$$\bar{q}_D = \frac{1}{\bar{\psi}_{rD}s^2} = \frac{1}{[A_2 I_0(r_{wD}\sqrt{\omega s}) + B_2 K_0(r_{wD}\sqrt{\omega s})]s^2} \tag{21}$$

V. DISCUSSION

Fig. 2 shows the comparison of dimensionless production rate type curves of transient dual dual-porosity model on log-log plot with or without radial flow effect. According Bello's research, a skin effect is introduced to interpret gas converge effect in the near-wellbore-zone and modify early stage type curves of horizontal wells in shale gas reservoirs. They concluded that as skin increases, only initial productivity decreases. However, in

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this paper, the results show that the radial flow effect not only results in severe reduction of early period production performance, but also the late period production performance.

Fig. 3 shows the effect of radial flow radius for dimensionless production rate for transient dual dual-porosity model on log-log plot. A larger radius denotes a larger gas pressure drop in the near-wellbore-zone for radial flow and a lower dimensionless production rate of multistage hydraulically fractured horizontal wells in shale gas reservoirs.

Fig. 4 shows the effect of fracture permeability for dimensionless production rate for transient dual dual-porosity model on log-log plot. The production rate in early period does not reduce with fracture permeability because matrix permeability is so low that fracture can be considered as infinite. However, when fracture permeability increase, the production rate in intermediate-late period is reducing because the time pressure drawdown reach the matrix center is shorter.

Fig. 5 shows the effect of fracture length for dimensionless production rate for transient dual dual-porosity model on log-log plot. Fracture length has no effect on multistage hydraulically fractured horizontal wells productivity in shale gas reservoirs. However, a longer fracture denotes a larger dimensionless production rate in intermediate-late period.

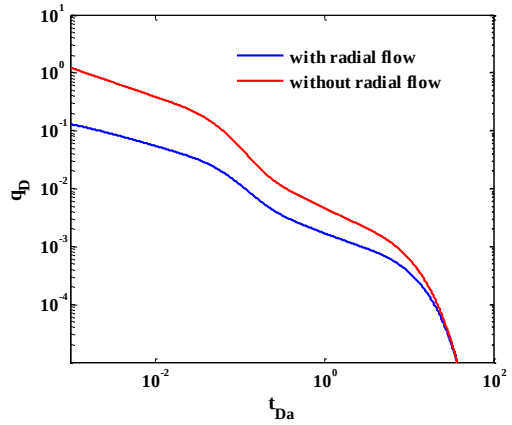

Figure 2. Type curves of slab transient dual porosity model with or without radial flow effect

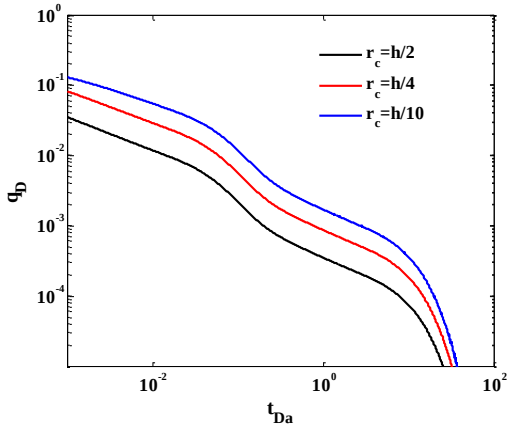

Figure 3. The effect of radial flow radius for dimensionless production rate

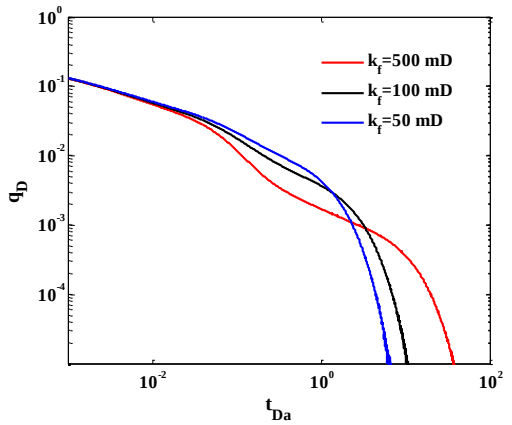

Figure 4. The effect of fracture permeability for dimensionless production rate

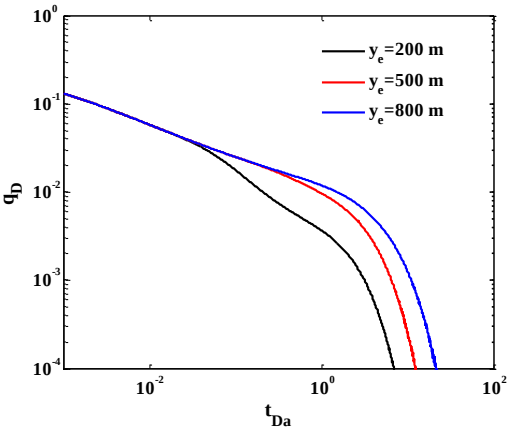

Figure 5. The effect of fracture length for dimensionless production rate

VI. Conclusion

In this paper, a fully transient dual-porosity model is established to describe three flow systems: linear flow from matrix to natural fractures, linear flow from natural fractures to near-wellbore-zone; radial flow from near-wellbore-zone to wellbore. The results show that the radial flow effect not only results in severe reduction of early period production performance, but also the late period production performance. Moreover, the production rate in early period does not change with fracture parameters because matrix permeability is so low fracture can be considered as infinite.

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