Scheduling Aluminum Billet Casting Lines: A Case Study

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Abstract—Driven by a real-world application in the aluminum industry, this paper provides a mathematical model to tackle the billet scheduling problem at the casting stage. The casting stage is where the molten metal is poured into the moulds to solidify and form the required shape based on the orders' specifications. The objective is to find an optimal scheduling for the billet production at the casting stage by minimizing the total production time and total set up times on the casting centers. In the aluminum industry, casting is a determinant of plant throughput as the production is limited by the amount that can be cast. So it is essential for the company to minimize setup time and processing time on production lines for a given time period to accommodate potential new orders. The problem is identified as a parallel machine scheduling problem with sequence-dependent setup times with very specific constraints imposed by the process control department to guarantee good quality products. A mathematical formulation to minimize the total processing and set up times on the casting centers is presented. Even though the problem is NP-hard, the model is solved using an exact method on a real case within just few seconds. Experimental results based on randomly generated data sets show that the model is efficient for most instances with reasonable sizes.

Index Terms—scheduling, mixed integer linear programming, aluminum production

I. INTRODUCTION

Globally, aluminum is a growing sector and is considered to be the metal of the future, especially due to its properties like lower weight, corrosion resistance and higher electrical conductivity [1]. Today, aluminum ranks number two in the consumption volumes among all the metals, surpassed only by steel. In the coming decades the demand for aluminum will continue increasing at unstoppable rates [2]. According to UC RUSAL, leader of the global aluminum industry (2015), overcapacity in the Chinese aluminum market continued throughout the first half of 2015. Thus, there is supply problem of raw material in this sector and today's competition in has made industries seek for more efficient and effective production planning methods. In Aluminum industry, the production process is very complex and involves a lot of characteristics such as casting, rolling and homogenizing. This constitutes a challenge for the planning systems where certain planning and scheduling principles are applied [3]. This research work is based on a real industrial case from a company competing in the aluminum industry. The focus is to define a short term production plan to a series of casting lines for the billet product.

Billets are a length of metal with round cross section (cylindrical shape) used for extrusion and forging processes in industry. The billet production process is done in two stages: casting stage and finishing stage. At the casting stage, billets are formed through direct casting machines. Each casting unit is connected to three furnaces, one smelter and two holder furnaces. The smelter furnace has the ability to melt high amount of scrap while the holders are used to accommodate the direct hot metal poured from the crucibles. During the furnace preparation, a unique recipe of elements is added to the pure aluminum such as silicon, magnesium, copper and many others. This unique recipe is referred to as Alloy. Once the metal is ready in the furnace, it is poured into the casting table where it will be solidified in the required billet diameter size. Each diameter requires a different casting table with the required mould size. After solidification, the billet logs enter the finishing stage whereby the billets are taken from the table to the laydown area where an inspection test is performed to capture any defects in billets. After that the billet logs enter the homogenizers to improve the mechanical properties of the billets.

In our case study, casting is a determinant of plant throughput as the production is limited by the amount that can be cast. So it is essential for the company to minimize production time on production lines for a given time period to accommodate potential new orders.

The site contains a total of five direct casting machines. However they differ in size, capacity, and capability of producing the different diameters, making the decision of assigning jobs to machines critical. Each diameter requires a different casting table which contains the required mould size. Some diameters share the same table base while the moulds are changed based on the

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size. Thus, the preparation (setup) time is sequence dependent.

In addition to minimizing processing times and setup times, two additional conditions set by the process control department at the company must be considered. The first condition imposes a minimum number of hours for the diameter moulds to stay in production before other diameter moulds are installed. This is imposed to avoid misusing the moulds as each time the moulds are removed they are sent to the mould shop to be prepared for the next run. It's also to provide the mould shop team enough time to prepare the moulds required for the next Diameter. The second condition limits the number of drops casted continuously using the same diameter moulds. This limit is set to avoid any quality issues in the product. If the number of drops reached this fixed number, the diameter moulds must be changed.

The remainder of the paper is organized as follows. In Section II, we present a brief survey of relevant literature and highlight the contribution of our work. The mathematical programming formulation is presented in Section III. Section IV presents the computational analysis on real data and on randomly generated instances. Finally, conclusions and future research directions are presented in Section V.

II. LITERATURE REVIEW AND CONTRIBUTION

Very few studies on aluminum industry address the scheduling problem. In [4], the scheduling problem on a single casting machine was addressed. In their model, the jobs are already assigned to the machines, so the objective was to find the sequence in which to cast aluminum logs such that set up times are minimized. The problem was formulated as a traveling salesman problem and solved using a genetic algorithm. In [5], a scheduling problem for continuous aluminum casting lines was studied. Their problem is very close to the problem addressed here in the sense that they are solving a continuous aluminum casting scheduling problem with parallel machines. However, [5] just consider work balance as an objective, instead of makespan minimization (in our case) and don't consider the new case-specific constraints imposed by the company in our case. The authors proposed a four step algorithm to find a good feasible solution in a reasonable amount of time. They solved a set of asymmetric travelling salesman problems by a pairwise exchange heuristic and applied it on a real case.

Our problem can be considered as an extension of the unrelated parallel machine scheduling problem with sequence dependent setup times. Tens of thousands of papers have been published on scheduling problems since the 1950s, where the first systematic approach to scheduling was used in [6]. Several books and textbooks were also published on the subject, including, among others, [7] and [8]. Among the most comprehensive surveys on scheduling problems, especially those with setup times/costs and parallel machines are [6] and [9]. Considering setup time in scheduling problems makes considerable cost reductions when solving scheduling problems. In most industries, it is not reasonable to just ignore them. This is the case in the aluminum industry. Furthermore, in most foundries, there are several furnaces or casting machines working in parallel and one of the decisions that has to be taken by the planner is the assignment of orders/jobs to the different parallel resources. Scheduling problems with parallel machines have also been extensively studied in the scheduling literature (See [7] for a presentation of these problems).

Scheduling problems are usually classified as NP-hard in the strong even in the case of single machine problems with some optimality criteria (See [10]). Thus most solution methods developed for such problems are heuristic and metaheuristic algorithms. Examples of heuristics developed for scheduling problems with parallel machines are presented below. Vallada and Ruiz [11] proposed a genetic algorithm that includes a fast local search and a local search enhanced crossover operator to solve the unrelated parallel machine scheduling problem minimizing the makespan. A restricted Simulated Annealing was presented by [12] which incorporates a restricted search strategy to minimize the makespan. Arnaout et al., [13] presented an Ant Colony Optimization to solve the unrelated parallel machines scheduling problem minimizing the makespan. The performance of the method was evaluated by comparing its solutions to solutions obtained using Tabu Search and MetaRaPS (Metaheuristic for Randomized Priority Search). A clonal selection algorithm was presented in [14] to solve the problem of minimizing the makespan on unrelated parallel machines with sequencedependent setup times. The algorithm developed was compared directly to the genetic algorithm proposed by [11], to the Ant Colony algorithm by [13] and to the Simulated Annealing algorithm proposed by [12], which have the best performance for the instances available in the literature. The comparison shows that the clonal algorithm was more efficient. A recent study of Rosales et al. [15] suggested a new makespan linearization and proposed several mixed integer formulations for the unrelated parallel machine scheduling problem with sequence and machine dependent setup times and makespan minimization. A metaheuristic was also developed providing small deviations from optimal solutions in medium sized instances.

Only few studies have developed exact methods to solve scheduling problems unrelated parallel machines. As the problem is NP hard, optimal solutions are found only for instances with a small number of machines and jobs [14]. Martello *et al.*, [16] presented a Branch and Bound (B&B) algorithm operating in conjunction with a Lagrangian relaxation for the determination of lower bounds for problems in which the objective is to minimize the makespan. Liaw *et al.*, [17] propose a B&B algorithm to minimize the weighted sum of tardiness, presenting a function as a lower bound and a heuristic as an upper bound. The setup times between jobs are not considered in any of these works. Rocha *et al.*, [18] considered the setup times and presented a B&B algorithm for minimizing the weighted sum of tardiness while Tran and Beck [19] used a logic based Benders decomposition approach to minimize the makespan.

Real-world production scheduling problems often result in even more intractable models; only realistic modeling of the problem features can help managers in their decisions. Thus, developing specific algorithms to solve the problem in such a way that these algorithms are flexible enough to accommodate frequent changes in the problem description by the companies is very challenging. It is for this reason that the authors have chosen to build a Mixed Integer Linear Programming formulation (MILP) that can be solved using a MILP solver and that can be modified easily to accommodate very specific constraints. The mathematical programming formulation is efficient in the sense that it needs very few minutes to find the optimal solution (see Section IV). Once the problem is well defined and all constraints are identified, it will be possible to think about other extensions of this study by developing fast and dedicated algorithms, for example.

III. PROBLEM FORMULATION

In order to state the scheduling model minimizing the total processing time and total set up times on the casting centres, the following notation is used: *Indices and sets:*

- n: number of jobs to be scheduled
- D: number of diameters than can be manufactured
- K: number of machines available
- i, j: Job (i, j = 1, ..., n)
- d: Job diameter (d = 1, ..., D)
- k: Machines (k = 1, ..., K)
- J_k: Set of jobs that can be processed on machine k *Parameters:*
- q_i: Size of job i
- d_i: Diameter of job i
- AV_k: Availability of machine k in hours based on shutdown plan
- p_{ik} : Processing time of job i on machine k
- S_{ijk}: Setup time required if job i immediately precedes job j on machine k
- MRH_d: Minimum run hours of diameter d
- Bs_{dk}: Average batch size of diameter d on machine k
- MaxC: Maximum number of batches allowed in one diameter before changing to another

Decision Variables:

- $Y_{ik} = 1$, if job i is assigned to machine k, 0 otherwise.
- $X_{ijk} = 1$, if job i is the immediate predecessor of job j on machine k, 0 otherwise.
- $Z_{dk} = 1$, if diameter d is assigned to casting line k, 0 otherwise
- U_{ik} : Continuous variable used in the subtour elimination constraints.

Using the above notation, a mixed integer linear programming model can be formulated as follows.

Minimize
$$\sum_{k} \sum_{i \in J_k} P_{ik} Y_{ik} + \sum_{k} \sum_{i \in J_k} \sum_{j \in J_k} S_{ijk} X_{ijk} \quad (1)$$

subject to

$$Y_{ik} = 0 \qquad \qquad \forall k, \forall i \notin J_k \quad (2)$$

$$\sum_{k \in K} Y_{ik} = 1 \qquad \qquad \forall i \neq 0 \qquad (3)$$

$$\sum_{i \in J_k} P_{ik} Y_{ik} + \sum_{i \in J_k} \sum_{j \in J_k} S_{ijk} X_{ijk} \le AV_k \quad \forall k \qquad (4)$$

$$Y_{0k} = 1 \qquad \qquad \forall k \quad (5)$$

$$\sum_{i \in J_k} X_{0ik} = 1 \qquad \forall k \quad (6)$$

$$\sum_{j \in J_{k}} X_{ijk} \leq Y_{ik} \qquad \forall k, \forall i \in J_{k} \quad (7)$$

$$\sum_{j} X_{jik} \leq Y_{ik} \qquad \forall k, \forall i \in J_k \quad (8)$$

$$\sum_{i \in J_k \setminus d_i = d} Y_{ik} \le n \times Z_{dk} \qquad \forall d , \forall k \qquad (9)$$

$$n \times \sum_{i \in J_k \setminus d_i = d} Y_{ik} \ge Z_{dk} \qquad \forall D, \forall k \quad (10)$$

$$\sum_{i \in J_k \setminus d_i = d} Y_{ik} P_{ik} \ge MRH_d Z_{dk} \qquad \forall d , k \qquad (11)$$

$$\sum_{i \in J_k \setminus d_i = d} \frac{q_i}{Bs_{dk}} Y_{ik} \le MaxC \qquad \forall d, k \qquad (12)$$

$$U_{ik} \geq P_{ik}Y_{ik} \qquad \forall i, i \neq 0, \forall k \qquad (13)$$

 $U_{ik} \leq AV_k \qquad \forall i, i \neq 0, \forall k \qquad (14)$

 $U_{ik} \ - \ U_{jk} \ + AV_k \ X_{ijk} \le \ AV_k - P_{jk} Y_{jk}$

 $\forall i \setminus i \neq 0, j, i \neq j, k \qquad (15)$

$$X_{ijk} \in \{0,1\}$$
 $\forall i, j, k$ (16)

$$Y_{ik} \in \{0,1\}$$
 $\forall i,k$ (17)

$$Z_{dk} \in \{0,1\} \qquad \forall d,k \qquad (18)$$

$$U_{ik} \ge 0 \qquad \forall i,k \qquad (19)$$

The objective function (1) of the proposed model is to minimize C_{Total} the total processing time and setup time of all jobs on all machines. Constraints (2) impose that each job i will not be assigned to machine k if it is not applicable on it (i $\notin J_k$). Constraints (3) ensure that all the jobs are assigned to a casting centre. Constraints (4) state that the workload assigned to any line cannot exceed the number of hours during which the machine is available for production. This also helps in considering the planned shutdowns for the casting centres. By applying constraints (5) and (6), we ensure that the

dummy job "0" is assigned to all machines to define the status of the machines while assigning the new jobs. Constraints (7) and (8) control the inflow and outflow of the jobs and ensure that a job succeeds/precedes only one job. Constraints (9) and (10) are added to define the value of Z_{dk} as a function of Y_{ik}. If any job of diameter d is assigned to machine k, then Z_{dk} must be = 1, 0 otherwise. Constraints (11) restrict the minimum hours for the diameter moulds to stay in production before another diameter moulds are installed. This is imposed to avoid misusing the moulds as each time the moulds are removed they are sent to the mould shop to be prepared for the next run. It's also to provide the mould shop team enough time to prepare the moulds required for the next Diameter. Constraint (12) guarantees that the number of drops casted continuously using the same diameter moulds does not exceed MaxC drops. This limit is set by process control to avoid any quality issues in the product. For example, in the test case, MaxC=80 drops. After the 80th drop the diameter moulds should be changed. Constraints (13), (14), and (15) represent the Miller-Tucker-Zemlin (MTZ) formulation. This formulation is added for sub-tours eliminations and it's inspired from [20]. Constraints (16) to (19) are integrality and nonnegativity constraints.

IV. COMPUTATIONAL ANALYSIS

On average the aluminum plant of our case study receives about 1000-1500 orders each month. Each order is characterized by its diameter, alloy and quantity. The company offers 13 different diameters and produces hundreds of different alloys that are grouped into families based on their chemical composition. A decision supported by the company management was made to further classify them based on their homogenizing route (Batch or Continuous). This decision was as the alloy type is not a significant criterion while scheduling the orders on the machines as the diameter size. This decision has identified the job in our model as group of orders that shares the same diameter and homogenizing route. For example, diameter A can have two jobs as A-Batch and A-Normal. As a result of this order consolidation, the number of jobs to be scheduled is reduced to 50-60 per month.

Т	ABLE	I. TEST	CASE	1 Data

Job	Diameter	Processing Time
1	1	40.70
2	1	12.21
3	2	81.40
4	2	28.49
5	3	46.33
6	3	28.30
7	4	56.98
8	4	36.63
9	5	56.43
10	5	48.84
11	6	40.70
12	6	44.77
13	7	116.80
14	8	71.50
15	9	38.80

A sample of consolidated data is given in Table I. The data concerns a weekly schedule. These data provide information on the diameter and processing time required for each job. The jobs are associated with 9 different Diameters. The 5 machines are operating with full capacity of 168 hours. The minimum run hour MRH is equal to 48 for all diameters except for diameter 9 which is equal to 24. The maximum limit drops is equal to 80 for all diameters.

The optimal solution is found in 1.5 seconds with the objective value of 761.49. The generated schedule is shown in "Fig. 1". The setup times are dark shaded in the figure.



Figure 1. Gantt chart for the company schedule weekly data

A. Experimentation on Randomly Generated Data

The model was implemented on FICO Xpress 2013, with optimizer version 24 running on Microsoft Windows 7 on a PC with CPU Intel i7 and 16GB RAM. It was tested on randomly generated problems. Sensitivity studies on number of jobs, number of diameters, availability of machines and density of the distribution of jobs on machines are also conducted, which provide some useful insights for industrial managers. Moreover, the proposed model is applied to a real-world case on a real data.

TABLE II. CPU FOR THE DIFFERENT TESTING SAMPLES WITH DIFFERENT AVAILABILITY OF MACHINES

		Without Shutdown	With Shutdown
	20	0.534	0.381
Jobs	30	2.457	1.781
	40	63.700	34.798
	80	218.140	151.308
	7	72.000	102.883
Diameter	15	22.116	39.532
	Average	63.158	55.114

Different samples of different sizes shown in Table II are tested for this computational study. For each problem size we studied the effect of the availability of the machines. We generate samples where all machines are functional (no planned shutdown) and samples where a shutdown occurs on one machine reducing its availability by 30%. In Table III, we studied also the effect of the distribution of jobs on machines as the machines are not identical and each one has its set of jobs that can process. We assume that in average jobs can be processed by 70% of the machines (density 70%), or 50% (density 50%). The results are summarized in Table III. For each case, five instances are generated and the average is calculated. The criterion used to measure and evaluate the effectiveness of the developed model is the CPU

computational time. It measures computation speed of the algorithm.

 TABLE III. CPU FOR THE DIFFERENT TESTING SAMPLES WITH

 DIFFERENT DENSITY OF DISTRIBUTION

		Density 70%	Density 50%
	20	0.7168	0.170
Jobs	30	63.0146	2.048
	40	6.2158	63.994
	80	144.4946	135.483
	7	95.887	63.678
Diameter	15	11.334	37.169
	Average	53.610	50.424

For all the samples tested, we can remark that the CPU is relatively small; the maximum is approximately 3min 30s for 80 jobs. With the growth of problem size, we can observe that CPU increases while in average CPU decreases when the number of diameters increases. This can be explained by the limitation of feasible solutions when the number of diameters increases. When varying the machine availability by considering the case with no shutdowns and the case with shutdowns on one machine reducing its availability by 30%, the results shown in Table II reveal that when shutdowns occur the program has less difficulty to find the optimal solution. However the CPU time remains small. From Table III, it can be noticed that the CPU time is very sensitive to the density of distribution of assignment of jobs to machines.

V. CONCLUSION

In this paper, a mathematical model for scheduling aluminum billet on casting centers in an aluminum industry located in Dubai was developed. The proposed model takes into consideration all requirements and constraints which are related to the casting stage to provide the best allocation and sequencing of jobs on the casting centers. The model was implemented and solved using a state of the art commercial solver where the optimal solution was found within seconds for real problem size. The provided model will be used by the company as initial step toward automating the whole planning and scheduling process of the company. Despite the accomplishment done, there are areas of improvement for the model and room for future work. The model presented here considers the casting stage only. It's recommended to include the homogenization stage and see what impact it will have on the schedule and total production output. It is also suggested to solve the model for a longer planning horizons. For a larger planning horizon, the problem become more challenging as the number of jobs increases and the complexity of the problem too. For these problems, we are intending to implement efficient metaheuristics to obtain good solutions within a very short CPU time.

REFERENCES

- J. R. Davis, *ASM International*, Illustrated edition, 1993, ch1.
 [Online]. Available:
- http://www.aluminiumleader.com/economics/world_market

- [3] F. David, H. Pierreval, and C. Caux, "Advanced planning and scheduling systems in the aluminum conversion industry," *International Journal of Computer Integrated Manufacturing*, vol. 19, no. 7, pp. 705-715, 2006.
- [4] P. Jensson, B. Kristinsdottir, and H. Gunnarsson, "Optimal sequencing of tasks in an aluminum smelter casthouse," *Computers in Industry*, vol. 56, pp. 221-234, 2005.
- [5] E. Duman, M. Yildirim, and A. Alkaya, "Scheduling continuous aluminum casting lines," *International Journal of Production Research*, pp. 5701-5718, 2008.
- [6] A. Allahverdi, C. Ng, T. Cheng, and M.Y. Kovalyov, "A survey of scheduling problems with setup times or costs," *European Journal of Operations Research*, vol. 187, pp. 985–1032, 2008.
- [7] M. L. Pinedo, "Scheduling: Theory, algorithms, and systems," Springer Science & Business Media, 2012.
- [8] P. Brucker, Scheduling Algorithms (3), Berlin: Springer, 2007.
- [9] A. Allahverdi, "The third comprehensive survey on scheduling problems with setup times/costs," *European Journal of Operational Research*, vol. 246, no. 2, pp. 345-378, 2015.
- [10] B. Chen, C. N. Potts, and G. J. Woeginger, "A review of machine scheduling: Complexity, algorithms and approximability," in *Handbook of combinatorial optimization*, Springer US, 1998, pp. 1493-1641.
- [11] E. Vallada and R. Ruiz, "A genetic algorithm for the unrelated parallel machine scheduling problem with sequence dependent setup times," *European Journal of Operations Research*, vol. 211, pp. 612–622, 2011.
- [12] K. C. Ying, Z. J. Lee, and S. W. Lin, "Makespan minimization for scheduling unrelated parallel machines with setup times," *Journal* of Intelligent Manufacturing, vol. 23, pp. 1795–1803, 2012.
- [13] J. P. Arnaout, R. Musa, and G. Rabadi, "A two-stage ant colony optimization algorithm to minimize the makespan on unrelated parallel machines: Part II: Enhancements and experimentations," *Journal Intelligent Manufacturing*, vol. 25, no. 1, pp. 43–53, 2014.
- [14] R. O. M Diana, M. F. F. Filho, S. R. D. Souza, and J. F. D. A. Vitor, "An immune-inspired algorithm for an unrelated parallel machines' scheduling problem with sequence and machine dependent setup-times for makespan minimization," *Neurocomputing*, vol. 163, pp. 94–105, 2015.
- [15] O. A. Rosales, F. Angel-Bello, and A. Alvarez, "Efficient metaheuristic algorithm and re-formulations for the unrelated parallel machine scheduling problem with sequence and machinedependent setup times," *The International Journal of Advanced Manufacturing Technology*, vol. 76, no. 9, pp. 1705-1718, 2015.
- [16] S. Martello, F. Soumis, and P. Toth, "Exact and approximation algorithms for makespan minimization on unrelated parallel machines," *Discrete Appl. Math.*, vol. 75, pp. 169–188, 1997.
- [17] C. F. Liaw, Y. K. Lin, C. Y. Cheng, and M. Chen, "Scheduling unrelated parallel machines to minimize total weighted tardiness," *Computers and Operations Research*, vol. 30, pp. 1777–1789, 2003.
- [18] P. L. Rocha, R. G. Ravetti, G. R. Mateus and P. M. Pardalos, "Exact algorithms for a scheduling problem with unrelated parallel machines and sequence and machine-dependent setup times," *Computers & Operations Research*, vol. 35, no. 4, pp. 1250–1264, 2008.
- [19] T. Tran and J. Beck, "Logic-based benders decomposition for alternative resource scheduling with sequence dependent setups," in *Proc. 20th European Conference on Artificial Intelligence*, 2012, pp. 774–780.
- [20] Brahimi and Aouam, "Multi-Item production routing problem with backordering: A comparative study of MILP formulations," Technical Report, University of Sharjah, 2012.

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