Lattice Boltzmann Simulation of Fluid Flow in Complex Porous Media Based on CT Image

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Abstract—In this paper, we simulate single phase and two-fluid-phase flow in realistic porous media using a lattice Boltzmann (LB). We used X-ray tomographic sandstones. These are binary digital models describing the complex pore space. First we show lattice boltzmann simulation results of single phase flow. We calculate the flow field and permeability of the micromodel and find agreement with experiment. Second, we simulate two-fluid-phase flow at the pore scale using the Shan-Chen model lattice boltzmann method. First, we calculate relative permeability for each fluid. Then we observe that the relative permeability of non-wetting phase increases with pressure gradient, while that of wetting phase stays almost the same.

Index Terms—lattice boltzmann, porous media, simulation, relative permeability

I. INTRODUCTION

Fluid flow in porous media is of crucial important role in many technological and environmental processes, such as the recovery of hydrocarbons from oil reservoirs. To characterize fluid flow in porous media, the experimental analysis is often used. However it is difficult and time-consuming. More recently, to simulate flow at the pore scale, it is viable to use simplified representations of porous media, such as physical micromodels, which can be constructed in the form of pseudo-two-dimensional capillary networks. Especially, the lattice-Boltzmann method has shown great potential for solving the flow in complex geometries and it has been successfully used in the study of fluid flow in porous media at the pore scale. Reference [1], [2] simulate single phase by LBM and calculate the permeability of porous media. N.S. Martys and H. Chen [3] applied the LBM to simulate multicomponent flow in complex three dimensional geometries. Reference [4] introduced the first detailed survey of LBE theory and its major applications to date. There have been many studies on single phase and two-fluid-phase flow in porous media. X. Shan and H. Chen [5] discussed a LB model which has the capability of simulating multiphase and multicomponent immiscible fluids. Reference [6] quantitatively evaluated the capability and accuracy of the lattice Boltzmann equation (LBE) for modeling flow through porous media. M. Venturoli and E.S. Boek [7] compared two-dimensional and three dimensional lattice-Boltzmann simulations of fluid flow in a pseudo-2D micromodel. In two dimensions, lattice-Boltzmann simulations have been used to investigate viscous fingering of binary immiscible fluids. Reference [8] applied a reactive transport lattice Boltzmann model developed in previous studies to study the dissolution-induced changes in permeability and porosity of two porous media at the pore scale. Reference [9] performed two-phase flow simulation on digital rock and discussed the effect of different initial saturation distribution of two fluids and relative permeability under different pressure gradient. Jianhui Yang and Edouard S. Boek [10] compared three different multi-component Lattice Boltzmann models to explore their capability of describing binary immiscible fluid fluids. The multicomponent multiphase LBM [11] proposed by Shan and Chen (SC) is a more popular model to simulate two-fluid-phase. The LBM method was used to calculate the relative permeability of sandstone and compared with experiment [12]. LBM simulations can also calculate flow in realistic porous media and compute relative permeability [13], [14]. In addition, Reference [15], [16] discussed the equations of state in the Lattice Boltzmann models.

This paper is organized in the following way. Firstly, the LB method and the basic concept will be introduced. Secondly, the permeability of a 2D micromodel of sandstone will be calculated and the calculation will be compared with the experimental data. Besides, we present results of simulations of multicomponent fluid in porous media. During this process, the relative permeability curve of a sandstone porous medium based on a CT Image will be calculated. Finally, the effect of ration of pressure gradient and surface tension on relative permeability will be analyzed.

II. THE LATTICE BOLTZMANN METHOD

The lattice boltzmann method consists of two processes. The first process describes particle advection
and the second describes particle collision on a regular lattice. The lattice Boltzmann advection and collision can be written in the following form [15]:

\[ f_i(x+\bar{c}i\Delta t+\Delta t)-f_i(x,t) = -\frac{1}{\tau}(f_i(x,t)-f_{\text{eq}}(x,t)) \]  

(1)

where \( f_i(x,t) \) is the number density distribution in the \( i \) th velocity direction for the fluid at position \( x \) and time \( t \), and \( \Delta t \) is the time increment, and \( \tau \) is the relaxation time in the lattice unit, \( f_{\text{eq}}(x,t) \) is the corresponding equilibrium distribution function. For the LBM model simulated in this work, we used the D2Q9 lattice structure, where \( D \) is the dimension and \( Q \) is the number of velocity direction. Then \( f_{\text{eq}}(x,t) \) has the following form [15]

\[ f_{\text{eq}}(x,t) = \rho \psi \left[ 1 + 3\varepsilon_i \frac{\bar{c}}{c^2} + \frac{9(\varepsilon_i \cdot \bar{u})^2}{2c^2} - \frac{2(\bar{u} \cdot \bar{a})^2}{2c^2} \right] \]  

(2)

In the above equation, \( \varepsilon_i \) is the discrete velocities, which can be written as

\[
\varepsilon_i = \begin{cases} 
(0,0) & i = 0 \\
\left( \cos \frac{(i-1)\pi}{2}, \sin \frac{(i-1)\pi}{2} \right) & i = 1, 2, 3, 4 \\
\sqrt{2} \left( \cos \left( \frac{(i-1)\pi}{2} + \frac{\pi}{4} \right), \sin \left( \frac{(i-1)\pi}{2} + \frac{\pi}{4} \right) \right) & i = 5, 6, 7, 8
\end{cases}
\]  

(3)

\( \rho \) is the molecular mass, which is defined through \( \rho = \sum f_i \), and the fluid velocity was calculated after every time step at each in the fluid volume [16]

\[ \bar{u} = \frac{\rho \bar{u}}{\rho} = \sum_{i=1}^{8} \varepsilon_i f_i / \sum_{i=1}^{8} f_i \]  

(4)

To simulate two-phase flow in porous media through SC LB model, we need to consider the fluid-fluid interaction force and fluid-solid interaction force. In the S-C model [16], the interparticle forces \( F_{f-f}(x) \), the fluid-fluid interaction force on \( k \) th fluid at site \( x \) is the sum of the force between the \( k \) th fluid particle at site \( x \) and the \( k \)'th fluid particles at the sites \( x' \)

\[ F_{f-f}(x) = -\rho \psi(x) \sum_{x'} G_{\psi}(x,x') \psi(x')(x' - x) \]  

(5)

where \( \psi^k(x) \) is a function of local density and \( G_{\psi}(x,x') \) represents the strength of the interparticle force. In this paper, \( \psi^k(x) = \rho^k \) is used for simplicity.

The interaction force between the \( k \) th fluid at site \( x \) and the solid wall at site \( x' \) is defined as [16]:

\[ F_{f-s}(x) = -\rho \psi(x) \sum_{x'} G_{\psi}(x,x')(x' - x) \]  

(6)

where \( G_{\psi}(x,x') \) represent the interactive strength between the \( k \) th fluid and the solid phase. The parameter \( s \) is 1 for a solid and 0 for a pore.

III. SINGLE PHASE FLOW SIMULATION

We simulate the flow in a 2D micromodel of Berea sandstone, which is presented in Fig. 1. We discretize on a lattice and bit-map to create the matrix for the LB simulation. The permeability of a porous medium can be calculated by the empirical Darcy’s law.

It is well known that the flow rate is proportional to the driving force, the permeability of the medium and the dynamic viscosity of the fluid. Darcy’s law can be defined as the follows

\[ J = -\frac{k}{\mu} \frac{\Delta P}{L} = -\frac{k}{\mu} \left( \frac{P_i - P_o}{L} \right) \]  

(7)

where \( J \) is the rate per unit area of cross section, \( k \) is the permeability, \( \nabla P \) is the pressure gradient, \( \mu \) is the dynamic viscosity of the fluid, \( P_i \) and \( P_o \) are the pressures at the inlet and outlet, respectively, \( L \) is the distance between the inlet and outlet.
To simulate the flow, we used periodic boundary conditions at the inlet and outlet, and no-slip boundary condition at all other boundaries. We performed simulations by 5 different values of the body force. Finally, we got the permeability \( k = 417 \) mD, agreement with the experimental value (\( k = 445 \pm 35 \) mD) [13].

Fig. 2 shows the details of the flow in the Berea sandstone. We can see that the velocity under steady state is different at different field. The higher flow rates almost concentrate in the core throat. The lower flow rates almost concentrate in the pore. This phenomenon is consistent with the test [13].

IV. TWO PHASE FLOW SIMULATION

The relative permeability of a phase is given as a ratio of single-phase permeability at a given saturation to the absolute permeability of the phase for a fully saturated porous media. Reference [17], [18] gave some methods to compute relative permeability. In order to calculate the relative permeability of a phase using the LBM method, we adopt a method [17] of normalizing the total momentum of a phase at a given saturation to the single-phase momentum, the formulation can be given as

\[
k_w = \frac{\pi (S_w) \Delta P(S_w = 1)}{\pi (S_n = 1) \Delta P(S_n)}
\]

In (8), \( \pi \) denotes the total momentum of each phase at a given wetting-phase saturation \( S_w \), and it can be calculated according to (4).

Values of relative permeability for different saturation of wetting and non-wetting fluids are shown in Fig 3, from which we can see that the relative of non-wetting fluid is very high and almost no permeability for wetting fluid at low wetting fluid saturation, as we wanted. And at high wetting fluid saturation, there are still considerable amount of relative permeability of non-wetting phase.

Fig. 4 shows the relative permeability curves obtained for three different ratios. It can be observed that the change in the ratio of the surface tension with the pressure gradient does not produce much change in the relative permeability of wetting fluid. But, the relative permeability of non-wetting fluid is affected significantly by the increase of pressure gradient.

We study the effect of pressure gradient (\( \Delta P \)) and surface tension (\( \sigma \)) on relative permeability. The ratio of the pressure gradient and surface tension was defined as follows

\[ C = \frac{\Delta P}{\sigma} \]  

V. CONCLUSIONS

In this study, we performed single and two-phase flow simulation on Berea sandstone using LBM method. First, we calculate the single phase permeability, which is good agreement with experiment. And we show the details of the flow in the Berea sandstone. In addition, we simulate two-phase flow and get the relative permeability curves, we found that the increase of pressure gradient cause the increase of the relative permeability of non-wetting fluid. But it is no effect on wetting phase. Finally, the LBM is a viable numerical tool to simulate single and two-phase flow in complex porous media at the pore scale and investigate fluid properties.

REFERENCES


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