Optimization of Well Location, Type and Trajectory by a Modified Particle Swarm Optimization Algorithm for the PUNQ-S3 Model

Shuaiwei Ding, Hanqiao Jiang, Junjian Li, Guangwei Liu, and Lidong Mi
Key Laboratory of Petroleum Engineering of the Ministry of Education, China University of Petroleum, Beijing, China
Email: shwding@126.com

Abstract—Determining the optimum well location, type and trajectory in an oil field is a challenging work for reservoir engineers. The problem is more complicated due to the wide variety of possible well types that must be considered. In this paper, a general methodology for the optimization of well location, type and trajectory in the combination of a modified particle swarm optimization algorithm and a well optimization model is presented. The modified particle swarm optimization algorithm is first discussed, and then the optimization model of well location, type and trajectory is proposed, and finally the implementation of the method for different well placement optimization problems for PUNQ-S3 model including different drive methods and well types is described. The results show the modification of standard particle swarm optimization algorithm is effective and the type (vertical, horizontal and deviated well) and the number of wells should also be an optimization parameter. For PUNQ-S3 model, it is reasonable to use vertical wells instead of horizontal or deviated wells and it is not wise to use water injection wells in the early development stage due to the strength of the aquifer.

Index Terms—well location optimization, well type optimization, standard particle swarm optimization algorithm, modified particle swarm optimization algorithm, PUNQ-S3 model, reservoir simulation

I. INTRODUCTION

As we all know, the need to find answers for where to drill wells in reservoirs with the maximization of a performance index (i.e., net present value or cumulative oil production) is a key factor in new development projects. To the best of our knowledge, the first attempt to optimize the well placement problem mathematically was done by Rosenwald and Green in 1974 when they used mixed integer programming (MIP) to find the best well locations from predefined well sites [1]. Since then, well placement optimization kept evolving to more involved and sophisticated optimization models with different types of algorithms such as genetic algorithm (GA), simulated annealing, particle swarm optimization (PSO), and Hook Jeeves direct search, in which both stochastic algorithms and deterministic algorithms were used with the goal of finding optimal well locations [1]-[6]. However, most of the researchers are focused on the optimization of well placement, and there are a few of literatures [7], [8] that report how to optimized well type, location and trajectory because the problem is more complicated due to the wide variety of possible well types (i.e., vertical, horizontal, deviated, multilateral, injector or producer) that must be considered.

The objective of this paper is to introduce and apply a general procedure for the optimization of well location, type and trajectory. We consider the vertical, horizontal and deviated well. The optimization approach entails the application of a modified PSO algorithm (MPSO), along with a well optimization model, objective function and constrains, used in conjunction with a reservoir simulator.

This paper is organized as follows: we first describe, in Section 2, the standard PSO algorithm and modified PSO algorithm, respectively. Then, in Section 3, the implementation of MPSO for well placement optimization used in this work is presented. Well placement optimization problems of varying complexity are considered in Section 4. We conclude with a brief summary in Section 5.

II. MODIFIED PSO ALGORITHM

A. Standard PSO Algorithm (SPSO)

PSO is a stochastic global optimization method developed by Eberhart and Kennedy [9]. It is based on the social behaviors observed in swarms of animals. PSO uses a set of candidate solutions at each iteration. Each of these candidate solutions is called a particle and the collection of particles is called the swarm. Like GA, PSO is a population based algorithm.

In the PSO algorithm, particle swarm consists of M particles, and the position of each particle stands for the potential solution in n-dimensional space. The position of the particle i at n-dimension noted as $x_{i}=(x_{i1}, x_{i2}, \ldots, x_{in})$, and the speed of the particle i at n-dimension is $v_{i}=(v_{i1}, v_{i2}, \ldots, v_{in})$. The optimal position of the particle i in the n-dimension quantity of the swarm is $p_{i}=(p_{i1}, p_{i2}, \ldots, p_{in})$, namely $p_{best}$. The optimal position of the whole swarm is $g_{i}=(g_{i1}, g_{i2}, \ldots, g_{in})$, also noted as $g_{best}$.
Here we use the nonlinear adjustment strategy and the parameters which are varying these parameters the PSO algorithm can be made more explorative or more exploitative (meaning less exploration but faster convergence).

These three key parameters impact the so-called inertia, position of each particle change according the following equation:

$$v_i(t+1)=w v_i(t)+c_1 r_1 (p_i(t)−x_i(t))+c_2 r_2 (p_g(t)−x_i(t))$$ (1) 

$$x_i(t+1)=x_i(t)+v_i(t+1)$$ (2)

where \( t \) is the current number of iterations; and \( w, c_1, \) and \( c_2 \) are weights; and \( r_1 \) and \( r_2 \) are random numbers within the interval (0,1); and \( p_g \) is the optimal position of the whole swarm. In general, SPSO uses \( w=0.721 \) and \( c_1=c_2=1.93. \)

As we all know, the performance of the PSO algorithm depends to some extent on the neighborhood topology. Particle topologies or neighborhoods refer to the grouping of particles into subgroups. A particular particle can communicate and exchange information about the search space only with other particles in its neighborhood [4]. There are several types of PSO neighborhood topologies [10], [11], and in fact we choose the star topology which has only one neighborhood and each particle has a link to every other particle as the SPSO neighborhood topology.

However, Onwunalu and Durlofsky (2010) [9] stated that they observed the best performance using a random neighborhood topology, which leads to a “local best” PSO algorithm (rather than global best PSO). So the performance of MPSO and SPSO which uses “local best” neighborhood topology instead of “global best” will be compared in the optimization runs and results section.

B. Modified PSO Algorithm (MPSO)

The performance of PSO is dependent on three parameters which are \( w, c_1, \) and \( c_2 \) used in the algorithm. These three key parameters impact the so-called inertia, cognitive and social components of particle velocity. By varying these parameters the PSO algorithm can be made more explorative or more exploitative (meaning less exploration but faster convergence).

Here we use a modified PSO algorithm (MPSO) instead of SPSO algorithm to optimize well placement. The novel algorithm is proposed through the modification of the following two aspects [5].

1) Modification of the inertia weight factor

Inertia weight \( w \) is a factor which is related to the previous velocity. It performs the role of the balance between global search and local search which means larger \( w \) could strengthen the ability of global detection of PSO and smaller \( w \) could strengthen the ability of local search. Several scholars have done some research on the modification of the inertia weight factor [12]-[14]. Here we use the nonlinear adjustment strategy and the modification is as the follows:

$$w(t)=w_{\text{max}}-(w_{\text{max}}−w_{\text{min}})\log\left(1+\frac{t}{t_{\text{max}}}\right)$$ (3)

where \( t \) is the current number of iterations; and \( t_{\text{max}} \) is the maximum number of iterations. Here we use \( w_{\text{max}}=0.9 \) and \( w_{\text{min}}=0.2 \).

2) Modification of the update strategy of velocity

In nature, the flight way of geese is the character of “person” glyph or “one” glyph which is very efficient and the flight distance could be increased by 71% than single goose flight [15]. When the geese fly, the leading wild goose in the flock could generate a trailing vortex by flapping its wings, and the companions flying behind could use this force to fly, so the leading wild goose is the most laborious and strongest one in the geese.

Using the characteristics of the flight of geese for reference, the strong degree of the geese could be regarded as the quality of the history optimal adaptive value of particles. We can sort all particles according to the quality of the history optimal adaptive values and choose the particle which has the best optimal adaptive value as the leading wild goose and put the others into the back of the sequence.

Thus, a modified PSO algorithm is proposed. For one thing, \( p_g(t) \) which is the optimal position of the whole particle swarm in Eq.(1) is replaced with \( p_{i-1}(t) \) which is the individual optimal position of the anterior goose, which means we regard the individual optimal position of the anterior goose as the global optimal position of the goose who flies behind it and the global optimal position of the leading wild goose is its own which will remain unchanged. In the PSO algorithm, this change will make the swarm diverse by re-ordering all particles and making each particle fly following its anterior particle.

For another thing, \( p_i(t) \) which is the optimal position of the particle \( i \) in Eq.(1) is replaced with \( p_{ad} \) for all particles except the leading wild goose. \( p_{ad} \) is a new individual optimal position from the weighted average of all particles’ individual optimal position and individual current adaptive value. This change means we regard that the leading wild goose makes decision only relying on its own experience, and the others behind it make decisions not only relying on their own experiences (individual optimal position) but also the experience from other geese. It strengthens cooperation and competition between particles by making each particle share more useful information from the other particles.

Finally, we could change the update strategy of velocity Eq.(1) as follows:

$$v_i(t+1)=w v_i(t)+c_1 r_1 (p_{ad}(t)−x_i(t))+c_2 r_2 (p_{i-1}(t)−x_i(t))$$ (4)

$$p_{ad}(t)=\frac{\sum_{i=1}^{N} p_i(t) \times f(x_i)}{\sum_{i=1}^{N} f(x_i)}$$ (5)

where \( p_{ad}(t) \) is the new individual optimal position of particle \( i \); and \( p_{i-1}(t) \) is the new global optimal position of particle \( i \); and \( f(x_i) \) is the individual current adaptive value of particle \( i \) which is also the objective function value (i.e. NPV in this paper).
III. IMPLEMENTATION OF MPSO FOR WELL PLACEMENT OPTIMIZATION

A. Establishment of Optimization Model of Well Trajectory

Here we establish an optimization model of well trajectory which can optimize three types of wells involving vertical, horizontal and deviated well.

For the optimization of well location and well trajectory of deviated well, the optimization model of well trajectory is shown in Fig. 1. In Fig. 1, the well location \((h_x, h_y, h_z)\) and well trajectory \((I_w)\) can be determined as long as the heel \((h_x, h_y, h_z)\) and toe \((t_x, t_y, t_z)\) is determined. In fact, the optimization of well location and well trajectory of vertical and horizontal well is two special cases of deviated well, which means the well is vertical if \(h_x = h_y = t_y\), and the well is horizontal if \(t_z - h_z = 0\).

In the PSO algorithm, the only thing we need to do is to use two particles in three-dimensional space to represent coordinates of heel \((h_x, h_y, h_z)\) and toe \((t_x, t_y, t_z)\) of the well to be optimized, then iterative optimization of objective function in the three-dimensional space can be automatically implemented by programming.

![Figure 1. Optimization model of well trajectory.](image)

B. Initialization of Well Optimization Model

The initialization of well optimization model in the optimization procedure is to choose the initial position of the particles, and here we use stochastic way to generate the initial position of the particles. For the optimization of vertical well, the only thing is to initialize the coordinate of \(x, y\) in two-dimensional space since it is not need to consider the optimization of well trajectory. However, for the optimization of horizontal and deviated well, the coordinates of heel \((h_x, h_y, h_z)\) and toe \((t_x, t_y, t_z)\) need to be initialized simultaneously according to the optimization model of well trajectory.

C. Objective Function Evaluation

In all problems considered, the net present value (NPV) is used as the objective function. Evaluating the NPV of each potential solution requires performing a simulation run. The resulting fluid production profiles generated from the simulation run are used to compute the NPV as follows [4], [5]:

\[
\text{NPV} = \sum_{t=1}^{T} \left( \frac{R_t - E_t}{(1+r)^t} \right) C^{\text{capex}}
\]

(6)

\[
R_t = p_o Q_t^p
\]

(7)

\[
E_t = p_w Q_{w,t}^{w,p} + p_i Q_{i,t}^{w,i}
\]

(8)

\[
C^{\text{capex}} = \sum_{w=1}^{N_{\text{well}}} \left( C_{w,t}^{\text{top}} + C_{w,t}^{\text{junc}} + C_{w,t}^{\text{main}} + C_{w,t}^{\text{drill}} \right)
\]

(9)

where \(T\) is the total production time in years; \(r\) is the annual discount rate; \(R_t\) and \(E_t\) represent the revenue ($) and operating expenses ($), respectively, at time \(t\); \(C^{\text{capex}}\) is the capital expenditure, which represents the total cost to drill and complete all of the wells; \(p_o\) represents the oil price ($/STB); \(Q_t^p\) represents the total volumes of oil (STB) produced at time \(t\); \(p_w\) represents water production costs ($/STB); \(Q_{w,t}^{w,p}\) represents the total volumes of water produced (STB) at time \(t\); \(p_i\) represents water injection costs ($/STB); \(Q_{w,t}^{w,i}\) represents the total volumes of water injected(STB) at time \(t\); \(N_{\text{well}}\) is the number of wells; \(C_{w,t}^{\text{top}}\) is the cost to drill the main bore to the top of the reservoir ($); \(C_{w,t}^{\text{junc}}\) is the single heel point cost of deviated or horizontal well ($); \(C_{w,t}^{\text{main}}\) is the length of the main bore (m); \(C_{w,t}^{\text{drill}}\) represents the drilling cost within the reservoir ($/m).

Here we take \(p_o\), \(p_w\), \(p_i\) and \(r\) to be constant in time in all cases. The economic parameters are given in Table I.

<table>
<thead>
<tr>
<th>Types</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling cost (to reservoir top) ($), (C_{w,t}^{\text{top}})</td>
<td>(50 \times 10^6)</td>
</tr>
<tr>
<td>single heel point cost ($), (C_{w,t}^{\text{junc}})</td>
<td>(15 \times 10^6)</td>
</tr>
<tr>
<td>Drilling cost per foot ($/m), (C_{w,t}^{\text{drill}})</td>
<td>(32803)</td>
</tr>
<tr>
<td>Oil price ($/STB), (p_o)</td>
<td>(45)</td>
</tr>
<tr>
<td>Water production cost ($/STB), (p_w)</td>
<td>(10)</td>
</tr>
<tr>
<td>Water injection cost ($/STB), (p_i)</td>
<td>(10)</td>
</tr>
<tr>
<td>Discount rate, (r)</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

D. Constrains and Treatment of Infeasible Particles

There are also some constraints in the optimization process:

First, make a hypothesis: \((x_i, y_i), (x_j, y_j)\) noted as the location of well \(i\) and \(j\), \(i,j = 1,2,\cdots, N_w\), \(i \neq j\). The following is the constraints:

Minimum distance between wells:
The coordinate of wells should be integers:
\[ x_i, y_i \in \mathbb{Z} \quad (11) \]

All wells should be in active grid cells: \( x_i, y_i \in \text{Active} \). 

Active is a matrix that stores coordinates of active grid cells.

For boundary-constrained optimization problems, direct application of Eq. (6) above may cause some particles to leave the feasible region of the search space. In addition, the well trajectory may intersect in the process of optimizing deviated or horizontal well. To handle these infeasible solutions, we introduce the penalty measures; that is if the position of the particles is located in the invalid grid or well trajectory is intersected by any two wells, we assign the adaptive value of these particles to be zero.

IV. APPLICATIONS

The PUNQ-S3 model has been taken from a reservoir engineering study on a real field. Many researchers have adopted this model to illustrate their reservoir theories [16]-[22]. The model contains 19×28×5 grid blocks, of which 1761 blocks are active. The top structure map shows that the field links to the north and west with a fairly strong aquifer. A small gas cap is located in the center of the dome shaped structure. Fig. 2 shows the three dimensional model. Owing to the strength of the aquifer, original well spacing scheme only has six vertical production wells, and injection wells is not required for pressure maintenance.

A. Case 1-Original Vertical Well Spacing Scheme

In case 1, in order to prove the superiority of well placement optimization, the optimization of original six vertical well spacing scheme is implemented using MPSO and vertical well optimization model. In this example, the swarm population size and maximum number of iterations are 50 and 100, respectively. These values were chosen based on suggestions in Alander [23]. Fig. 3 shows a comparison of NPV as a function of the total number of iterations for original well spacing scheme and well spacing scheme by MPSO. For each of the example problems, we repeat the optimization runs three times because of the stochastic nature of PSO algorithm, and the solid blue line corresponds to the average MPSO procedures, and the dotted blue lines correspond to the three MPSO procedures, and the solid red line corresponds to the original well spacing scheme.

From Fig. 3, it is evident that the MPSO based well spacing scheme has the absolute superiority when compared with the original well spacing scheme. From Fig. 3, it is presented that the average MPSO based well spacing scheme and original well spacing scheme give an NPV of \(2.0 \times 10^8\) and \(1.46 \times 10^8\), respectively. The result has been improved by 37.37%, so the MPSO method with vertical well optimization mode is effective for PUNQ-S3 reservoir with depletion-drive development.

B. Case 2-Deviated Production Well Spacing Scheme

In case 2, in order to prove the effectiveness of the modification of SPSO and superiority of MPSO with deviated well optimization model, the optimization of location and trajectory of six deviated production wells is implemented using MPSO and SPSO together with deviated well optimization model. In this example, the swarm population size and maximum number of iterations are 50 and 100, respectively. Fig. 5 shows a comparison of NPV as a function of the total number of iterations for MPSO and SPSO. For each of the example problems, we repeat the optimization runs three times because of the stochastic nature of PSO algorithm, and the solid blue line corresponds to the average MPSO
procedures, and the dotted blue lines correspond to the three MPSO procedures, and the solid red line corresponds to the average SPSO procedures, and the dotted red lines correspond to the three SPSO procedures.

From Fig. 5, it is evident that the MPSO based well spacing scheme has the absolute superiority when compared with the SPSO based well spacing scheme. From Fig. 5, it is presented that the average MPSO and SPSO based well spacing scheme give an NPV of $1.37 \times 10^8$ and $1.19 \times 10^8$, respectively. The result has been improved by 15.55%, so the modification of SPSO method is effective for PUNQ-S3 reservoir with depletion-drive development.

![Figure 5. NPV as a function of the total number of iterations for MPSO and SPSO for case 2.](image)

When we compared the highest NPV achieved by MPSO and SPSO, it is noted that the value of case 2 is lower than that of case 1. However, the oil production of case 2 is higher than case 1, and the water production of case 2 is lower than case 1 when we compared the liquid production between each other. The reason behind this phenomenon is that the basic investment cost is increased using horizontal and deviated wells which lead to the economic profit of case 2 is lower than that of case 1. So it can be seen, the use of vertical wells is better than the use of horizontal and deviated wells for the development of PUNQ-S3 reservoir, and the number of wells should be decreased if the horizontal and deviated wells are chosen.

![Figure 6. A comparison of well placements proposed by MPSO and SPSO methods for case 2 (Solid black circles correspond to production wells and solid green circles correspond to the trajectory of the wells).](image)

The optimal well locations for the highest NPV case using MPSO and SPSO procedures are shown in Fig. 6. It is noted that the optimization result of well types of both procedures are horizontal and deviated wells, the producers are still located around the oil-gas interface, but the well spacing is more uniform and the length of the horizontal well is more short by MPSO.

C. Case 3-Deviated Injection and Production Well Spacing Scheme

In case 3, similarly, in order to prove the effectiveness of the modification of SPSO and superiority of MPSO with deviated well optimization model, the optimization of location and trajectory of one deviated injection well and five deviated production wells is implemented by using MPSO and SPSO together with deviated well optimization model. In this example, the swarm population size and maximum number of iterations are still 50 and 100, respectively. Fig. 7 shows a comparison of NPV as a function of the total number of iterations per realization for MPSO and SPSO. For each of the example problems, we repeat the optimization runs three times because of the stochastic nature of PSO algorithm, and the solid blue line corresponds to the average MPSO procedures, and the dotted blue lines correspond to the three MPSO procedures, and the solid red line corresponds to the average SPSO procedures, and the dotted red lines correspond to the three SPSO procedures.

![Figure 7. NPV as a function of the total number of iterations for MPSO and SPSO for case 3.](image)

From Fig. 7, it is evident that the MPSO based well spacing scheme also has the absolute superiority when compared with the SPSO based well spacing scheme. From Fig. 7, it is presented that the average MPSO and SPSO based well spacing scheme give an NPV of $1.2 \times 10^8$ and $1.09 \times 10^8$, respectively. The result has been improved by 11.47%, so the modification of SPSO method is effective for PUNQ-S3 reservoir with depletion-drive development.

When we compared the highest NPV achieved by MPSO and SPSO, it is noted that the value of case 3 is lower than that of case 2. However, the oil production of case 3 is almost same as case 2 when we compared the liquid production between each other. The reason behind this phenomenon is that the water injected costs is increased through early water injection which lead to the economic profit of case 3 is lower than that of case 2. So it can be seen, the use of water injection well is not reasonable for the development of PUNQ-S3 reservoir,
and it is suggested to add one or two water injection wells in the later period.

The optimal well locations for the highest NPV case using MPSO and SPSO procedure are shown in Fig. 8. It is noted that the optimization results of MPSO based well spacing scheme are horizontal and deviated wells, the producers are still located around the oil-gas interface, but the injection well is vertical well and is close to the oil-water interface in Western of the reservoir. Meanwhile, the optimization results of SPSO based well spacing scheme are horizontal and deviated wells, all the wells are located around the oil-gas interface, but the injection well is horizontal well and is located in the center of the reservoir.

V. CONCLUSIONS

In this paper, we proposed a method which combined a modified particle swarm optimization algorithm and an optimization model of well trajectory to optimize well placement. The modified PSO algorithm is proposed through the modification of the inertia weight factor and the update strategy of velocity.

In order to prove the best results of our method, we considered a variety of example problems, involving original vertical well spacing scheme, deviated production well spacing scheme, and deviated injection and production well spacing scheme for PUNQ-S3 model, with the maximization of NPV as the objective function. In all cases considered except case 1, we demonstrated that the MPSO based method outperforms SPSO based method, which proves the modification is effective and the method performs the best.

In case 2 and 3, we optimized well placement for PUNQ-S3 model with different driving methods and different well types, the results suggest that the type (vertical, horizontal and deviated well) and the number of wells should also be an optimization parameter. For PUNQ-S3 model, the use of vertical wells is better than the use of horizontal and deviated wells and the use of water injection wells is not reasonable.

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Shuaiwei Ding PhD student in China University of Petroleum, Beijing, China, expect to receive PhD degree of Oil and Gas Field Development Engineering in June 2016. His main research area is reservoir engineering, reservoir numerical simulation, well placement optimization and profile control. He is an author or co-author of over 11 papers. Dr. Ding is also a student member of Society of Petroleum Engineers.

Hanqiao Jiang Professor in China University of Petroleum, Beijing, China. His main research area is reservoir engineering, water shutoff, profile control, enhanced oil recovery and reservoir numerical simulation. He is an author or co-author of over 200 papers. Now he is a Chief of National Basic Research Program of China “973” funded by Ministry of Science and Technology run by five universities and research institutions.

Junjian Li Assistant Professor in China University of Petroleum, Beijing, China, receiving PhD degree in 2010. His main research area is reservoir engineering, water shutoff, polymer flooding, enhanced oil recovery and reservoir numerical simulation. He is an author or co-author of over 50 papers. Dr. Li is also a member of Society of Petroleum Engineers.

Guangwei Liu PhD student in China University of Petroleum, Beijing, China, expect to receive PhD degree of Oil and Gas Field Development Engineering in June 2016. His main research area is reservoir engineering, reservoir numerical simulation and water control in horizontal wells. He is an author or co-author of over 7 papers. Dr. Liu is also a student member of Society of Petroleum Engineers.

Lidong Mi PhD student in China University of Petroleum, Beijing, China, expect to receive PhD degree of Oil and Gas Field Development Engineering in June 2017. His main research area is reservoir engineering and shale gas development. He is an author or co-author of over 10 papers. Dr. Mi is also a student member of Society of Petroleum Engineers.