# Evaluate the Performance of a Hybrid Manufacturing System Using GI/G/1 Network

Ping-Yu Chang

Department of Industrial Engineering and Management, Ming Chi University of Technology, New Taipei City, Taiwan Email: pchang@mail.mcut.edu.tw

Abstract—This paper proposes an Queueing network to investigate the impact of a hybrid manufacturing system. The hybrid manufacturing system is a new conceptual manufacturing system that contains multiple channels that are analogous to cells of CM but where each channel resembles a JS configuration. Moreover, each channel contains the required machine types but fewer machines of each type than JS. Also, each product will always be processed in the same channel in terms of no inter-channel movements in MCM system. There are two objectives for this research. The first objective is to develop queueing approximations of flow time and WIP level for the hybrid system. In particular, GI/G/c approximations are developed using open queueing network in this research. The second objective is to evaluate the impact of the key factors on the performance of the hybrid system. By changing the value of the key factors in the approximations, the insights for the system performance can be realized. Through analytical models, different factors are analyzed and a suitable manufacturing system design with the combination of the factors is identified.

*Index Terms*—hybrid manufacturing system, open queueing network, GI/G/c

## I. INTRODUCTION

In industry today, manufacturing systems of different designs have been used. The choice of the manufacturing system design depends on the demand, product types, and process characteristics. For example, when the demand includes a large variety of different parts with different process sequences, a job shop (JS) will often be an appropriate design. Besides the characteristics of product and process, a good manufacturing system should be able to response to changes, such as demand fluctuations. In particular, a good manufacturing system should have flexibility to adjust in the uncertain environments.

To meet these challenges, a new manufacturing system called Multi-Channel Manufacturing (MCM) has been introduced by Meller [1]. MCM is based on the simple observation that an effective manufacturing system provides multiple channels for each manufactured product as it flows through the system, i.e., the system provides more than one channel or path for the products to go through the system. The goal of MCM is to decrease the throughput time, inventory, and material

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handling, while increasing flexibility, throughput, line-ofsight management, and effective space utilization. However, its usefulness in industry has not yet been studied as MCM research is still in its beginning stages. It is important and pertinent for this research to investigate MCM in order to understand the advantages and disadvantages of this manufacturing configuration strategy.

This research can be briefly summarized as follows. The literature review is presented in Section 2. The strategies and methodologies in creating the analytical models are discussed in Section 3. In Section 4, the applicability of the methodology is tested with numerical examples. Finally, Section 5 provides conclusions, contributions, and future directions for this research.

# II. LITERATURE REVIEW

Analytical models are an abstraction of a real system in terms of quantitative relationships described by mathematical expressions [2]. An analytical model contains a mathematical expression that allows the manufacturing system to be easily interpreted and understood. In this research, the mathematical expression of manufacturing systems is obtained by adopting queueing theory.

From the literature, Suresh [3] was the first to compare Cellular Manufacturing (CM) and Job Shop (JS) using analytical models. He considered a work center with c similar machines that are partitioned into a system with each machine dedicated to subfamilies of parts. The unpartitioned system is analyzed using a M/M/c model and the partitioned system is analyzed using c singleserver, M/M/1 models. Suresh presented a numerical example, the results of which showed that with a specified flow time or WIP level, the machine utilization may be lower in the partitioned system. The setup reduction in the partitioned system has to deteriorate the performance of the CM system before benefiting performance. The result also showed that for a given lot size, the performance of a partitioned system with the setup reduction factor (the multiplier between 0 and 1 of the setup time) greater than the setup reduction factor at breakeven point will be inferior to the corresponding unpartitioned system.

Kannan and Palocsay [4] used queueing theory to examine the relationship between processing time learning rates and flow time performance in CM and JS. Their results showed that it is possible for a CM to outperform a JS but CM required a higher learning rate to be able to compete with JS.

Buzacott and Shanthikumar [5]-[7] reviewed the use of stochastic models of manufacturing systems. In particular, their research concentrated on the flow line, transfer line, FMS, JS, and CM. They developed mathematical approximations for throughput, cycle time, and work-inprocess in different manufacturing scenarios. These approximations are examined through examples to the best approximation for different classify manufacturing scenarios. For example, thev recommended the GI-arrival approximation for any type of JS, whereas M-arrival approximations are recommended for large randomly routed JS ([8], [9]).

In the literature review, it has been shown that both simulation models and analytical models facilitate the analysis and comparison of the performance of manufacturing systems. Simulation models and analytical models will be used to better understand the MCM system and facilitate comparison of MCM to other manufacturing systems, in particular CM and JS.

# III. HELPFUL HINTS

The analytical models here are developed to verify the influence of the key parameters. Before introducing the approximations, some notation for MCM is provided.

N: number of workstations,

I: number of types of product,

*h*,*j*,*l*: index of workstations,  $h,j,l = \{1,2,\ldots,N\}$ ,

*i*: index of products,  $i = \{1, 2, \dots I\}$ ,

 $E[S_{il}]$ : expected service time for product *i* at station *l*,

 $C^{2}[S_{il}]$ : SCV (squared coefficient of variation) of

service time for product *i* in station *l*,

 $p_{hj}^{i}$ : probability for product *i* to travel from station *j* to station *h*.

*k*: number of channels,  $a = \{1, 2, \dots, K\}$ ,

 $\lambda_{hi}^{k}$ : arrival rate of product *i* at station *h* in channel *k*,

 $\gamma_{hi}$ : external arrival rate of product *i* to station *h*,

 $P_{lj}^k$ : routing probabilities from station *l* to station *j* in channel *k*,

 $\lambda_{i}^{k}$ : total arrival rate of station *j* in channel *k*,

 $\gamma_{i}^{k}$ : total external arrival rate of station j in channel k,

 $m_l^k$ : number of machines in station *l* in channel *k*,

 $U_l^k$ : utilization of station *l* in channel *k*,

 $P_{ki}$ : probability that product *i* is assigned to channel *k*,  $C_a^2(k, j)$ : SCV of inter-arrival time for station *j* in channel *k*,

 $C_s^a(k, j)$ : SCV of service time for station *j* in channel *k*, CTs(k,j): cycle time of station *j* in channel *k*,

E[Ts(k,l)]: expected service time of station *l* in channel *k*, WIP(i): work-in-process of product *i*.

Two critical assumptions are made to develop the queueing approximations.

- (1) The inter-arrival times are independent and identically distributed, which would represent a renewal process.
- (2) The branch decision is an independent random draw for each job, called a Markovian routing so that the resultant splitting is again a renewal process.

Following Curry [10] and Buzacott and Shanthikumar [7], the notation and approximations will be addressed further in this section.

The queueing approximations for MCM are developed from the generalized open queueing network. The WIP for each product in each channel is accumulated and becomes the WIP level for each product in the system. The cycle time is then determined using Little's Law. An additional assumption for developing the approximations is that no inter-channel movements are allowed in the system.

The inflow for each workstation can be computed using

$$\lambda_{hi}^{k} = P_{ki}\gamma_{hi} + \sum_{j=1}^{N} p_{jh}^{i}\lambda_{hi}^{k}, \qquad \forall h, i, k.$$
(1)

The inflow given by (1) is then used to develop  $\lambda_j^k$ and  $P_{lj}^k$  that are computed by using (2) and (3) while the total external rate for the station,  $\gamma_j^a$ , is computed by using (4).

$$\lambda_j^k = \sum_{i=1}^I \lambda_{ji}^k \qquad \forall j, k,$$
<sup>(2)</sup>

$$P_{lj}^{k} = \frac{\sum\limits_{i=1}^{l} p_{lj}^{i} \lambda_{li}^{k}}{\lambda_{l}^{k}} \qquad \forall j, l, k,$$

$$(3)$$

$$\gamma_j^k = \sum_{i=1}^{I} P_{ki} \gamma_{ji} \qquad \forall j, k.$$
<sup>(4)</sup>

The inflow is used to determine the SCV of the arrival process and the SCV of the service process as shown in (5) and (6).

$$C_{a}^{2}(k,j) = \frac{\sum_{i=1}^{l} \gamma_{ij} C_{a}^{2}(0,i)}{\lambda_{j}} + \sum_{i=1}^{N} \frac{\lambda_{i}^{k} P_{ij}^{k} [P_{ij}^{k} ((1 - (U_{i}^{k})^{2}) C_{a}^{2}(k,l) + (U_{i}^{k})^{2} C_{s}^{2}(k,l)) + 1 - P_{ij}^{k}]}{\lambda_{j}^{k}} \qquad \forall j,k,$$
(5)

$$C_{s}^{2}(k,l) = \frac{\sum_{i=1}^{l} \frac{\lambda_{l_{i}}^{i}}{\lambda_{l}^{k}} E[S_{il}]^{2} (1 + C^{2}[S_{il}])}{(\sum_{i=1}^{l} \frac{\lambda_{l_{i}}^{k}}{\lambda_{l}^{k}} E[S_{il}])^{2}} - 1 \qquad \forall l,k,$$
(6)

where 
$$U_l^k = \frac{\sum\limits_{i=1}^{l} \lambda_{li}^k E[S_{il}]}{m_l^k}, \quad \forall l, k.$$
 (7)

The cycle time at each workstation in each channel and the WIP for each product are then determined using (8) and (10).

$$CTs(k,j) = \left(\frac{C_a^2(k,j) + C_s^2(k,j)}{2}\right) \left(\frac{(U_j^k)^{\sqrt{2m_j+2}}}{m_j(1-U_j^k)}\right) E[Ts(k,j)] + E[Ts(k,j)] \quad \forall j,k,$$
(8)

where 
$$E[Ts(k,l)] = \sum_{i=1}^{l} \frac{\lambda_{li}^k}{\lambda_l^k} E[S_{il}] \qquad \forall l,k,$$
 (9)

$$WIP(i) = \sum_{k=1}^{K} \sum_{j=1}^{N} CTs(k, j)\lambda_{ji} \qquad \forall i.$$
(10)

## IV. RESULTS AND COMPARISONS

The effect of four factors, number of channels, scheduling strategies, batch size, and number of material handling vehicles (e.g. forklifts, or automated guided vehicles), on system performance is studied in this research. The applicability of the developed models is tested on an example derived from Abdelmola et al. [11]. This manufacturing system consists of eleven parts and seven machines. The setup time is assumed to be 3 hours for each machine. To sum up the factorial design, the levels of each factor are shown in Table I.

TABLE I. FACTORIALS OF EXPERIMENTAL DESIGN

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		# Of challets							
# of vehicles	Batch Size\scheduling rules	1		2		3		СМ	
1	10	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
	30	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
	50	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
2	10	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
	30	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
	50	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
3	10	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
	30	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT
	50	FIFO	SPT	FIFO	SPT	FIFO	SPT	FIFO	SPT

The experimental design is followed, and MATLAB is used for solving the analytical models. The inputs to the analytical models, such as channel assignment probability and material handling time, are evaluated in the simulation models. The results provide WIP levels for evaluating the cost function of the manufacturing systems.

Fig. 1 shows the effect of the number of material handling vehicles on flow time. For example, when there is one material handling vehicle in the system, the flow time in 2-channel MCM system is around 38 minutes and the flow time in 3-channel MCM is around 37 minutes. Fig. 1 indicates that as the number of material handling

vehicles increases, the flow time will decrease in all manufacturing systems. A significant decrease in flow time is realized when the number of vehicles increases from 1 to 2. The results demonstrate that the increase in material handling vehicles will result in the decrease in flow time. However, MCM outperforms JS when the material handling system is no longer the bottleneck, which is because of shorter material handling time in MCM.



Figure 1. Flow time vs number of material handing vehicles

The phenomenon also results from not including machine breakdowns. MCM will be less flexible than JS when machine breakdowns are considered. To improve MCM performance when machine breakdowns are considered, the design should allow inter-channel movements (i.e., allow product to move to another channel whenever a machine breaks down). This design requires a mechanism for moving and scheduling products. State dependent analysis would also have to be added to the analytical model. This extension represents a potential area for further research.



Figure 2. Flow time vs batch size

Fig. 2 shows the effect of increasing batch sizes on system performance. For example, the flow time in 2-channel MCM system is around 36 minutes and the flow time in 3-channel MCM system is around 35 minutes when the batch size is 10. The results indicate that the

batch size has no impact on flow time when the number of vehicles is two and the scheduling rule is FIFO. These results indicate that all manufacturing systems approach a performance plateau where increasing batch size no longer improves the flow time. The analysis and comparison of all manufacturing systems through analytical models indicate influence of key factors in which MCM may achieve better flow times. When there are fewer material handling vehicles available in the system. MCM outperforms JS and CM. Designing the best MCM, however, will depend on balancing the trade off between the cost of increasing one channel and the effect of introducing an additional channel on the flow time. Adding one more channel will result in shorter flow time when the material handling time is critical, but may increase machine cost. Increasing batch size and changing the scheduling rules will shorten the flow time for MCM but the change is not significant. When material handling time is not significant and the setup time reduction is possible, MCM again outperforms JS. The magnitude of the performance difference depends on the length of setup time and the percentage of setup time reduction.

### V. CONCLUSION AND FUTURE RESEARCH

This research investigates MCM in two areas: classification and robustness. In particular, this research tries to identify the impact of the key factors on MCM performance. Through analytical models, different levels of key factors are analyzed and a suitable combination of key factors for which MCM can achieve better performance is identified. The analytical results illustrate that with limited material handling capacity and opportunities for setup time reduction, MCM can outperform JS and CM. The analytical results also show that the flow time in MCM varies only slightly with different batch sizes, whereas the flow time in JS and CM differs significantly with different batch sizes when material handling capacity is limited.

Using MCM can be beneficial to industries with a large variety of products, seasonal products, or short product life cycle (e.g., computer accessories and toy manufacturers). More specifically, using MCM reduces the material handling time so that the cycle time can be improved for companies with large varieties of product. Also, MCM is better able to adjust to varying conditions,

such as changes in the number of material handling vehicles due to preventive maintenance or equipment breakdowns. This paper builds a foundation on MCM research that can be used for further investigation in which different dispatching rules and a specific material handling system can be applied to MCM.

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**Ping-Yu Chang** is an Assistant Professor in the Department of Industrial Engineering and Management at Ming Chi University of Technology (MCUT), Taiwan. He received his master degree in Manufacturing Engineering at Syracuse University in 1996 and his Ph.D. degree in Industrial Engineering at Texas A&M University in 2002. His current research and teaching interests are in the Supply Chain and Production

Management. In particular, he is interested in Supply Chain Management, Facility Location, Scheduling, and Simulation Modeling.