A Modified Mixed Integer Programming Model for Train Rescheduling

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***Abstract*—Mathematical optimization techniques have been widely used in modeling and solving rail transportation problem. In dealing with conflicting trains during service disruptions, rescheduling train aims to produce an adjusted periodic timetable for the affected trains using available resources while satisfying a set of operational constraints. In this paper, we present some modifications on a mixed integer programming (MIP) model with the objective of minimizing the total service delays when service disruptions occur. Based on a selected reference model, the sets, parameters and the decision variables of the modified model are thoroughly discussed in this paper. Two problem cases with small sample data were created to test the model and interpret the reschedule results. The solutions that have been generated successfully provide the new provisional timetable, indicating the total delay experienced by trains.**

***Index Terms*—mathematical optimization model, mixed integer programming, service delays, railway rescheduling**

I. INTRODUCTION

Operational problems and unexpected events such as technical failures, equipment breakdown, extraordinary passenger volumes, track accidents or weather conditions normally cause disruptions to railway network. In this situation, control managers need to reshuffle train orders, make unplanned stops and break connections, re-route trains and even delay or cancel scheduled services. Changes in the original train departure and arrival schedules can create conflicts in the use of tracks and platforms. Thus, operational decisions must resolve the problem and reschedule the affected train movement with an objective to minimize the effect of railway traffic perturbations.

This paper intends to present some modification on a mathematical model for solving post-disruption railway rescheduling problem that minimizes the total delays of trains in the whole railway network. To achieve the objective, a new mixed integer programming (MIP) model for rescheduling railway is proposed. This paper illustrates the important elements for the model construction, including the sets, parameters and variables. It is also expected to highlight the future direction of the modeling works. The complete model and the solution approach are expected to bring new ideas for multiple perspective improvements in delay management, as well as business engineering process as well as quality engineering improvement.

This paper is outlined as follows: Section II discusses some relevant literatures on the railway rescheduling model construction. The strength and weaknesses of the reference model are briefly discussed in Section III, while Section IV highlights the crucial elements in the mathematical model construction. Section V presents the computational results while Section VI briefly explains the complexity of the model concerning the complicated combinatorial problem. The conclusion and further research of the study are drawn in Section VII.

II. RELATED WORKS

Among the various types of quantitative models used in rescheduling railway services, a study done by Alwadood, Shuib and Hamid [1] has shown that integer programming (IP) and MIP are widely used in formulating the optimization problem. They are technically chosen because the models are able to accommodate the linearity of the objective functions and constraints.

This section summarizes and compares the criteria that are relevant in the formulation of the mathematical models which are used in selected literatures of train rescheduling problem. Among the published results are the works of Narayanaswami and Rangaraj [2], Caimi Fuchsberger, Laumanns and Luthi [3], Acuna-Agost [4], Stanojevic, Maric, Kratica, Bojovic and Milenkovic [5], Murali [6], Afonso [7], Zhou and Zhong [8], Tornquist and Persson [9] and Tornquist and Persson [10].

Basically, the mathematical models are based on three sets namely train, block or segment and station, as shown explicitly in Table I (A). The set of trains contains all types of train running on the rail track which may be in outbound or inbound direction, or in some works it is known as up line or down line direction. The set of blocks or segments is the collection of all sections of railway tracks which can only be occupied by one train in a direction, at any particular time.

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A train is disallowed to enter an empty block section without first securing the permission of the station. The set of stations is the entire terminal meet point for the trains within the relevant study area. These three sets are dominant in all the models identified in the nine selected literatures. However the sets of events for each of these train, block or segment and station are only used in some of the studies. These sets of events are the resource requested by a specific train, block or segment and station, respectively.

TABLE I. CROSS ANALYSIS OF SETS, PARAMETERS AND DECISION VARIABLES USED IN MODELS OF RELATED WORKS

	AUTHORS								
(A) SETS	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
train	●	●	●	●	●	●	●	●	●
block/segment		●	●	●	●	●	●	●	●
station	●	●		●		●	●	●	
events for train			●					●	●
events for block/segment			●					●	●
events in stations			●						
(B) PARAMETERS	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
initial start of event of train as in timetable	●	●	●	●	●	●	●	●	●
initial end of event of train as in timetable	●	●	●		●	●	●	●	●
earliest start time of event of train	●	●	●		●			●	●
earliest end time of event of train	●	●	●	●				●	●
separation time for meeting trains	●		●			●	●	●	●
separation time for following trains	●		●			●	●	●	●
minimum running (waiting) time for event	●		●			●	●	●	●
penalty/cost per time unit for delays		●	●					●	●
large positive constant	●		●		●		●		●
time horizon/time index	●	●		●	●		●		
dwell time				●		●	●		
last event on a train			●	●					
next event on a train				●					
capacity of meetpoint			●	●	●	●			
direction of event of train	●	●	●				●	●	
length of each track					●			●	
length of each train								●	
(C) DECISION VARIABLES	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
start time of event for train	●	●	●	●	●	●	●	●	●
end time of event for train	●	●	●	●	●	●	●	●	●
event uses track *t* or not		●	●	●	●		●	●	
event on block occurs before or after an event			●				●	●	●
magnitude of delay for event for train	●	●	●					●	●
the order of events change or not	●		●					●	
an unplanned stop is added during event or not			●				●		
event uses train *h* or not				●	●				
train *h* is leaving or waiting at station *k*				●					

[1] Narayanaswami & Rangaraj, 2013; [2] Caimi et al., 2012; [3] Acuna Agost, 2010; [4] Stanojevic et al., 2010; [5]Murali, 2010, [6] Afonso, 2008; [7] Zhou & Zhong, 2007; [8] Tornquist & Persson, 2007; [9] Tornquist & Persson, 2005.

Due to the dimensions of the problem and the complex nature of the IP and MIP models, the selection of the parameters that would be taken into the model formulation needs to be closely examined. Table I (B) provides the cross analysis of some common main model parameters used in the selected research works. Many selected models share almost similar parameters but there are also models which incorporate a set of unique parameters as an attempt to improve an existing model or to introduce a hybrid mathematical model. For instance, Tornquist and Perrson [10] introduced the parameters for train connection in order to handle the objective function of costs for missing connections. In addition to this, Acuna Agost [4] introduced the parameters of braking and accelerating time as a result of unplanned stops.

Having said this, to come up with a newly-developed model, it is recommended that the common listed parameters should be first included to ensure the sensibility of the model. This will then be followed by introducing fresh elements in the model formulation to offer a unique research novelty.

As all the research works aim to arrive at provisional timetables which are able to minimize service delay, then it is expected that the most important decision variables in the model formulation should be the *start time* and *end time* of the event for train. Other decision variables relate to which train that is to be used, which track the train should run on, which station the train should be leaving from or waiting at, among others. For the decision variables involving *'yes'* or *'no'* answer, binary variables of *'1'* and *'0'* are used. The cross analysis of decision variables used in the mathematical programming models in the selected literatures is shown in Table I (C).

III. The Reference Model

Train rescheduling is a large size combinatorial problem. In many countries nowadays, the railway infrastructure is no longer operating on an isolated or separate railway tracks but rather involving high integration of rail network which consists of many interact railway lines. A cross analysis on the sets, parameters and decision variables that have been used in various mathematical models is done by Alwadood, Shuib and Hamid [11]. Among all the models analyzed, Tornquist and Persson [9] MIP model is selected to be the reference model for this work. The model aims to minimize the total final delay of the railway traffic and the total cost associated with delays. It is a strong formulation of rescheduling railway traffic problem in railway network because it accommodates the concept of multi-operator tracked lines, multi-passenger demands and highly interacting traffic. Other mathematical and non-mathematical models did not address these complexities in the model formulation. Therefore, this model has been chosen as the reference model in this study.

In spite of these strengths, few aspects are disregarded in the model formulation which may affect the practicality of the generated outcomes. For instance, due to the limited availability of data, all stations in the experimented rail network are assumed to have four parallel tracks. The assumption may be an oversimplification as stations could have far greater number of parallel tracks or only a minimum number of two tracks for smaller stations. Other flaw of the model is due to a constraint which indicates that two trains must be separated by certain time duration when they are on the same track of a segment. In practice, two trains using different tracks on different segments must also be separated by certain gap of time. One scenario would be when two trains using different tracks on different segments are going to cross each other and heading to the other track of the subsequent segments. Even when these two trains are using different tracks of a segment, the action can still violate the rail safety restrictions and may possibly cause an accident. To cater to the conflicting routing, the signaling switches between the tracks need to be included in the model formulation, so as to coordinate each event in a synchronized sequence and avoid accidents.

IV. The Modified Model

Based on the work done by Alwadood, Shuib and Hamid [11], the new mathematical model to be formulated will be using notation from Tornquist and Persson model, in addition to some selected parameters from other literatures including Stanojevic, Maric, Kratica, Bojovic and Milenkovic (2010), Murali (2010) and Afonso (2008). Hence some of the notations used for the preliminary model's sets, parameters and decision variables will be the same.

A. Sets and Parameters

Basically, the model is based on three sets. The first set is the set of trains T with index i, where $i \in T$ contains all train running on the rail track, which may be in up line or down line direction.

The second set is the set of segment B with index j, where $j \in B$ is the collection of all sections of railway tracks, which are separated by the series of signal switch control. An ordinary train route has a signal switch control located right before the block entrance. Once accepted, no other trains will be allowed on the block or segment without permission from the signal switch. The distance between the signals varies along the track based on the geographical safety aspects such as the elevation of ground and the blind spots at curving turn.

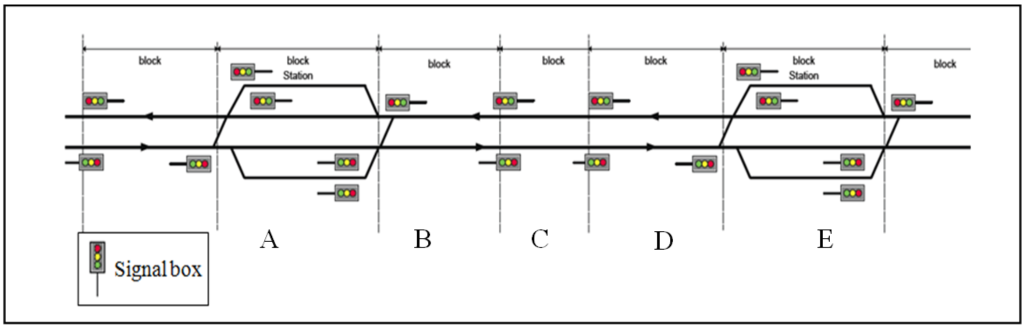

Figure 1. Types of segment

There are two types of segment location l which are the non-station segment and station segment. These are denoted by $l_j = 1$ and $l_j = 0$, respectively. Fig. 1 illustrates the segment between stations (B, C and D) and the segment within stations (A and E). For each segment, there is a set of parallel tracks $P_j = \{1, \ldots, p_j\}$.

A standard railway safety regulation normally imposes a minimum distance between two consecutive trains to avoid accidents. The distance which is usually termed as time headway, indicates the time when a train exits from a segment and the subsequent train enters the same segment. The parameter H_j denotes the time headway in case when one train is following the other on a track of segment j.

Finally, the third set is the set of events E with index k, where $k \in E$ are the resource requested by a specific train for a specific block. $K_i \subseteq E$ is the ordered set of events of train i, and $L_j \subseteq E$ is the ordered set of events of segment j, as established in the original timetable. For each of the set K_i and L_j, their last event in each set is denoted by n_i and m_j, respectively. The first proceeding event of event k is denoted by $(k+1)$. On the other hand, $\hat{k}$ is used to denote any event following event k, where $\hat{k} > k$.

Let r_k be the route of event k, having the value of *'1'* if it is in up line direction and *'0'* if it is in down line direction. For an event k running at a segment, the minimum time is denoted by the parameter Δ_k, which starts when the first train coach enters the segment. The speed is assumed constant, as the train speed profile including the rates of acceleration and braking are ignored in the model.

A train timetable defines the scheduled departure and arrival time to satisfy passengers' demand in reaching their destinations. The parameter d_k^S and a_k^S specify the scheduled departure and scheduled arrival time of event k as in timetable, respectively. In spite of the schedule established, the real-time train events do not always start and end as time planned. To take the possibilities of any instantaneous deviation into account, d_k^A and a_k^A will denote the actual departure and actual arrival time of event k.

M is an arbitrarily large positive constant. It is included so that a constraint is binding when a binary decision variable takes a value of 1. On the other hand, if it takes a value of 0, the constraint will become redundant. In addition to this, δ_i and η_i are two constants introduced to denote the time taken for a train to pass one non-station and station segment, respectively.

B. Decision Variables

The mathematical programming model aims to produce a provisional timetable when disruption occurs. There are seven decision variables anticipated from the solution method. The first two are d_k^R and a_k^R, which represent the start time and the end time of the rescheduled event k, where $k \in E$, $K_i \subseteq E$ and $L_j \subseteq E$. As a result of the new schedule, the amount of delay that will turn out from the rescheduling event k will be denoted by the decision variable z_k.

The assignment of trains to tracks is defined by:

$$tr_k^t = \begin{cases} 1, & \text{if event } k \text{ uses track } t, \\ & \text{where } t \in P_j, k \in L_j, j \in B, \\ 0, & \text{otherwise} \end{cases}$$

In order to determine if more than one train meet at the same segment, the following decision variable is defined:

$$s_{k\hat{k}} = \begin{cases} 1, & \text{if } (d_{\hat{k}}^R \;\; d_k^R \geqslant 0) \wedge (a_k^R \;\; a_{\hat{k}}^R \geqslant 0), \\ & \text{where } k, \hat{k} \in L_j, j \in B, k < \hat{k} \\ 0, & \text{otherwise} \end{cases}$$

When $s_{k\hat{k}} = 1$, we define f_s and f_l to be the first and the last concurrent event in the segment, respectively.

In some cases of disruption, event k may need to be rescheduled to occur after the event $\hat{k}$. Therefore, in order to determine the order of the events, the common scheduling disjunctive binary decision variables $\gamma_{k\hat{k}}$ and $\lambda_{k\hat{k}}$ will be used:

$$\gamma_{k\hat{k}} = \begin{cases} 1, & \text{if event } k \text{ occurs before event } \hat{k}, \\ & \text{where } k, \hat{k} \in L_j, j \in B, k < \hat{k} \\ 0, & \text{otherwise} \end{cases}$$

$$\lambda_{k\hat{k}} = \begin{cases} 1, & \text{if event } k \text{ occurs before event } \hat{k}, \\ & \text{where } k, \hat{k} \in L_j, j \in B, k < \hat{k} \\ 0, & \text{otherwise} \end{cases}$$

The complete model formulation is given by:

$$\text{Minimize} \sum_{i \in T} z_{n_i} \tag{1}$$

Subject to

$$a_k^R \leqslant d_{k+1}^R, \quad k \in K_i \quad, i \in T : k \neq n_i \tag{2}$$

$$a_k^R \geqslant d_k^R + \Delta_k, \quad k \in E \tag{3}$$

$$a_k^R = d_{k+1}^R, \quad k \in K_i \quad, i \in T : k \neq n_i \tag{4}$$

$$a_k^R = d_k^R + \Delta_k, \quad k \in E \tag{5}$$

$$d_k^R \geqslant d_k^S, \quad k \in E \tag{6}$$

$$d_k^R = d_k^A, \quad k \in E \quad : \quad d_k^S > 0 \tag{7}$$

$$a_k^R = a_k^A, \quad k \in E \quad : \quad a_k^A > 0 \tag{8}$$

$$a_k^R - a_k^S \leqslant z_k \quad k \in E \tag{9}$$

$$\sum_{t=1}^{p_j} tr_k^t = 1, k \in L_j, j \in B \tag{10}$$

$$\sum_{k=f_s}^{f_l}\sum_{t=1}^{P_j} tr_k^t \leq P_j \;, t\in P_j, k\in K_i, j\in B \tag{11}$$

$$tr_k^t + tr_{\hat{k}}^t - 1 \leq \lambda_{k\hat{k}} + \gamma_{k\hat{k}} \;, \quad k,\hat{k}\in L_j \;, t\in P_j \;, j\in B : k<\hat{k} \tag{12}$$

$$\lambda_{k\hat{k}} + \gamma_{k\hat{k}} \leq 1, \; k,\hat{k}\in L_j \;, j\in B : k<\hat{k} \tag{13}$$

$$d_{\hat{k}}^R - a_k^R \geq H_j\gamma_{k\hat{k}} - M(1-\gamma_{k\hat{k}}) \quad k,\hat{k}\in L_j, j\in B : k<\hat{k}, o_{\hat{k}} = o_k \;, l_j = 0 \tag{14a}$$

$$d_{\hat{k}}^R - a_k^R \geq H_j\gamma_{k\hat{k}} - M(1-\gamma_{k\hat{k}}) \quad k,\hat{k}\in L_j, j\in B : k<\hat{k}, o_{\hat{k}} = o_k \;, l_j = 1 \tag{14b}$$

$$d_k^R - a_{\hat{k}}^R \geq H_j\lambda_{k\hat{k}} - M(1-\lambda_{k\hat{k}}) \;, \quad k,\hat{k}\in L_j, j\in B : k<\hat{k}, o_{\hat{k}} = o_k \;, l_j = 0 \tag{15a}$$

$$d_k^R - a_{\hat{k}}^R \geq H_j\lambda_{k\hat{k}} - M(1-\lambda_{k\hat{k}}) \quad k,\hat{k}\in L_j, j\in B : k<\hat{k}, o_{\hat{k}} = o_k \;, l_j = 1 \tag{15b}$$

$$d_{\hat{k}}^R, a_k^R, z_k \geq 0, k\in E \tag{16}$$

$$tr_k^t \in \{0,1\}, k\in L_j, t\in P_j, j\in B \tag{17}$$

$$\gamma_{k\hat{k}} \;, \lambda_{k\hat{k}} \in \{0,1\}, k,\hat{k}\in L_j \;, j\in B : k<\hat{k} \tag{18}$$

$$tr_k^t \in \{0,1\}, k\in L_j \;, t\in P_j \;, j\in B \tag{19}$$

The objective function (1) calculates the minimum sum of delays experienced by all trains when they reach the final destination. Constraints (2)-(3) are constraints that govern the commuter trains. Constraint (2) indicates that a successor of a train event must wait until its predecessor has been completed, before it can start. The minimum running time for each train event is guaranteed by Constraint (3). Constraints (4)-(5) define the restriction posed for the prioritized train, Electric Train System (ETS). Constraint (4) ensures that each ETS train event must be directly succeeded by the next one, as far as the original schedule is concerned. Constraint (5) guarantees that the ETS trains should strictly depart and arrive, according to the planned scheduled.

Constraint (6) indicates that the reschedule departure time should never be earlier than the original time scheduled. Constraints (7)-(8) force new departure and arrival time in the occurrence of disruption. Constraint (9) defines the total delay of all trains as the deviation between the rescheduled and the original arrival times.

Constraint (10) restricts the utilization of track line as one train per track. Constraint (11) is introduced to ensure that the total concurrent events must not exceed the track capacity. Constraint (12) checks the order sequence between an event and its proceeding event, so as to ensure that it is either $\gamma_{k\hat{k}}$ or $\lambda_{k\hat{k}}$ will take value of ‘1’ in Constraint (13).

Constraints (14)-(15) impose a restriction for the minimum headway between two following trains. Two trains in the opposite direction are not considered in the constraint because although they are running or waiting at the same segment, these trains are never allowed to use the same track. Hence, imposing a minimum headway for them is unnecessary. It is either the set of Constraint (14) or Constraint (15) that will become active, depending on the value of $\gamma_{k\hat{k}}$ and $\lambda_{k\hat{k}}$. In addition to this, the minimum headway H_j equals $2\eta_i$ for a station segment and $3\delta_i$ for a non-station segment, respectively. For these sets of constraint, M is a large positive integer. Finally, the constraints (16)-(17) define the domain of the decision variables.

V. The Computational Results

The raw data obtained from a railway company in Malaysia has been used to serve as a sample data for the purpose of model testing. The railway line connecting some cities in the Klang Valley of Malaysia is presented in Fig. 2.

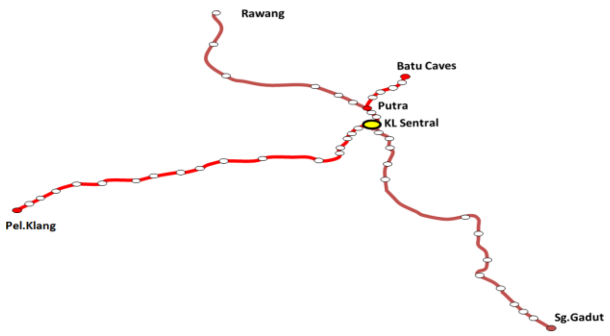

Figure 2. Rail network for a train services in the Klang Valley of Malaysia

From the original train schedules and the location of the signaling switches along the rail track, the data of time and segment is extracted to form a time-space diagram as shown in Fig. 3.

The entire data for the train service comprises of 120 service trips which runs on 67 rail track segments involving 25 stations. There are hundreds of events that lead to a mathematical model with thousands of variable and constraint.

In order to test the model and interpret the reschedule results, a preliminary experiment was run on a small sample of 9 segments, 5 stations and 5 running trains in up line and down line direction. Only Segments 8 and Segment 9 have four rail tracks while the rest are equipped with double track system. There are a total of 38 events and 229 decision variables altogether, which technically produce a list of 290 model constraints. Two types of trains are selected as a sample in the experiment, namely the Komuter train and the higher prioritized train, ETS.

Segment / Time	Station A 1	2	3	4	Station B 5	6	7	Station C 8	9	10	11	Station D 12	Station E 13	14	Station F 15
420	Train 12	Train 12						Train 9	Train 9						Train 1
421		Train 12							Train 9	Train 9					Train 1
422		Train 12	Train 12							Train 9	Train 9			Train 1	Train 1
423			Train 12	Train 12							Train 9	Train 9	Train 1	Train 1	
424				Train 12								Train 9	Train 1		
425				Train 12	Train 12							Train 9	Train 1/9		
426					Train 12								Train 1/9		
427					Train 12							Train 1	Train 1/9		
428					Train 12	Train 12						Train 1	Train 9	Train 9	
429						Train 12	Train 12				Train 1	Train 1		Train 9	
430							Train 12			Train 1	Train 1			Train 9	Train 9
431							Train 12	Train 12	Train 1	Train 1					Train 9
432								Train 1/12	Train 1						Train 9
433								Train 1/12							Train 9
434	Train 10							Train 1/12							Train 13
435	Train 10	Train 10					Train 1	Train 1/12	Train 12						Train 13
436		Train 10					Train 1		Train 12	Train 12					Train 13
437		Train 10	Train 10			Train 1	Train 1			Train 12	Train 12			Train 13	Train 13
438			Train 10	Train 10		Train 1					Train 12	Train 12	Train 13	Train 13	
439				Train 10	Train 1/10	Train 1						Train 12	Train 13		
440					Train 1/10							Train 12	Train 12/13		

Figure 3. A snapshot of the Malaysian train time-space diagram

Each train is assumed to be able to fit on any track and a maximum of six-car train is assumed. The location of all trains in the network is known at all times. For simplicity, the speed of trains is assumed constant and the dwell time of the trains at stations is embedded in the event duration.

The time scope is limited to 30 minutes and two sample problem cases were manually created to capture the rescheduling scenario. The cases were randomly selected based on the track capacity at the disruption site and the reaction of affected train towards maintaining the minimum headway. The mathematical model is solved using MATLAB R2011a. The computational tests were run on a 3.00GHz AMD Phenom Processor with 4Gb RAM.

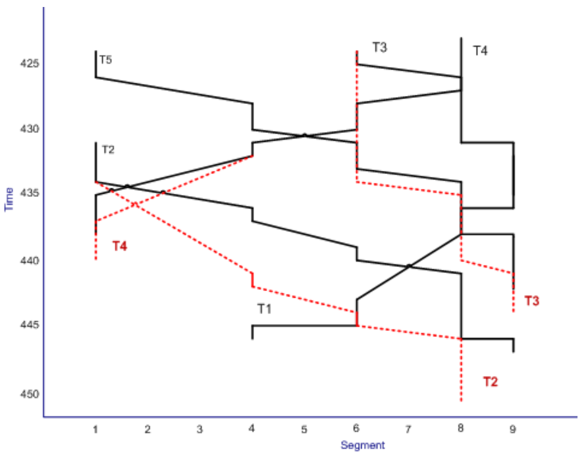

Figure 4. Optimal rescheduling for Case 1

Fig. 4 displays the movement of the five trains that take place in between segments, along the rail tracks. The vertical axis depicts the time while the horizontal axis shows the segment. Each line in the diagram represents a train, which is indicated by the number next to the line. Lines with positive gradient indicate the movement of train from segment 1 to segment 9. The lines with negative gradient indicate the opposite direction. Note that there are vertical lines at segment 1, 4, 6, 8 and 9, which imply that trains are dwelling for some time at stations.

A. Problem Case 1

Suppose Train 3 experiences a 9-minute delay at Segment 6. The dotted lines in Fig. 4 represents the optimal reschedule plan generated by the optimization model. In relation to this, Train 2 is forced to be delayed right from Segment 1 so as to meet the track capacity constraint at Segment 8. On the other hand, Train 4 is shifted as well to adhere to the headway restriction posed by the model.

The optimum solution resulted from the model shows that the minimum delays experienced by Train 2, Train 3 and Train 4 are 5 minutes, 9 minutes and 2 minutes, respectively. As a consequence, this has brought the objective function of total delay *z* to be 16 minutes.

B. Problem Case 2

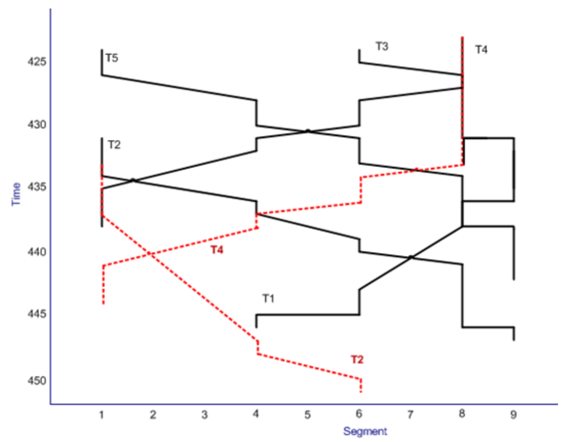

Figure 5. Optimal rescheduling for Case 2

Now suppose Train 4 experiences a 6-minute delay at Segment 8. Fig. 5 illustrates the new reschedule plan for the affected trains. Once it has overcome the problem, it starts heading to its destination, maintaining its normal pattern of running, However, Train 2 takes a longer time from Segment 1 to reach to the next station, due to the model restrictions on minimum headway. As a result, the Train 2 has changed sequence with Train 1 along segment 4 to segment 6, that changed the sequence. The computational result generated shows that the minimum delays incurred by Train 2 and Train 4 are 11 minutes and

6 minutes, respectively. To this end, the objective function yields the total delay z to be 17 minutes.

From the two problem cases that have been analyzed, the MIP model has successfully generates the rescheduling timetable. The total service delay for each affected train is attainable, together with the list of all departure and arrival times for each event. The new rescheduling plan does make sense, considering the results in the aspects of track capacity and the minimum headway. In addition to this, as a prioritized train, Train 5 which is an ETS, runs according to the planned schedule.

However, some aspects have been disregarded in this experiment. The scope of data used in this experiment might be too small. A more complicated result might be obtained if more segments are covered, more trains are included and the time horizon is lengthened. The possibility of train overtaking the other is also neglected; while in practice, a train is allowed to overtake the damaged train at a segment loop. The model formulation also disregards the need for connecting trains. In some railway system, a train is forced to wait for other train arrival to connect their passengers to their final destination. Finally, the ability of the chosen software to process a large scale data may need to be confirmed, before the experiment is carried out.

VI. The Complexity of the Model

As a preliminary experiment, this paper attempts to cover a small scale problem of railway rescheduling. Once the model is verified true, only then it can be run on a large scale data. This procedure must be implemented because railway rescheduling is a very complex task.

Railway rescheduling involves real-time alteration of train schedules in a railway network which is highly interconnected. Mathematically, this is considered as a difficult, combinatorial and strongly constrained problem. The model's constraints require a large number of hard (operational) constraints and soft (desirability) constraints and the complexity of problem increases with the number of decision variables and constraints. Modeling and solving this railway rescheduling problem is thus considered a highly complex task and an NP-hard problem.

Train rescheduling model needs to be run at macro level of railway networks so as to meet the real-world application demands. The routing and scheduling tasks are very challenging because it normally involves large combinatorial optimization problems. In the early stage, it demands the ability to formulate the real problem into a mathematical representation, incorporating all the factors influencing the decision variables, not forgetting the constraints and uncertainties governing the problem. In later stage, it demands the ability to solve the problem and generate the feasible or optimal solution within a short time frame, using search method or exact method, whichever suits the model.

The algorithm intends to solve railway traffic conflict as fast as possible so as to assist the dispatcher in the resolution process. Solution to conflicts may involve many combinations of stations, departure and arrival times, direction of routes and location of conflicts, especially when the disruption involves a train that interferes with other trains. Therefore, depending on the chosen solution for a conflict, optimal solutions are normally unattainable in large-scale and complex instances, besides the number of feasible solutions can be very large.

VII. Conclusion and Further Research

This paper presented a preliminary experiment done to a small scale railway system with the aim to minimize the total service delays. A modified MIP model for rescheduling has been used to be the tool in finding the optimal solution. By means of a base model, the main elements supporting the model construction have been discussed in detail, with some additional features being highlighted. Two problem cases were created and the MIP model is able to generate optimal solutions.

For the ongoing research, our model will be tested in a large scale setting and evaluated in a more practical context. The aspects that have been disregarded earlier, will be given a greater concern. The analysis will also be looking at some other problem cases, possibly involving signaling issue or route clash with ETS.

Heuristic computational methods using AMPL programming language and CPLEX Solver will next be implemented. The model is intended to produce quantifiable quick solution to the real-time rescheduling problem and offers service recovery strategies which help the railway services to maintain as an efficient and reliable mode of transportation.

The work is part of the study that is conducted on train rescheduling model to cope with the railway service disruptions within Malaysia commuter rail system. With the modified mathematical model, the solution approaches to the mathematical model will next be presented in next research papers of our study.

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