

# Multi-Depot Instances for Inventory Routing Problem Using Clustering Techniques

N. M. Noor and A. Shuib

Faculty of Computer and Mathematical Sciences, UiTM Shah Alam, Selangor, Malaysia

Email: {norlenda, adibah253}@salam.uitm.edu.my

**Abstract**—This paper proposes the development of multi depot test instances using clustering technique, which are single linkage and complete linkage clustering. Current benchmark instances for inventory routing problem are based on single depot. Instances for multi depot problem are still lacking. Originally, there are more than 100 datasets available with different number of customers for that were developed randomly. However, all the datasets are meant for single depot. In this paper, 45 sets of data containing 100 customers were used to create 15 test datasets. These test dataset will be used for validation purposes once the model has been developed. Sample results from these test instances are presented. Results show that multi-depot instances can be obtained and that clustering techniques implemented helps to group the data into the desired numbers of clusters.

**Index Terms**—inventory routing problem, benchmark instances, multi depot

## I. INTRODUCTION

Inventory routing problem (IRP) is a class of problem where, inventory and routing were solved simultaneously. Generally, basic version of IRP concerns with repeated distribution of an item from a single facility to a set of customer over a given planning horizon. A fleet of homogeneous vehicle with limited capacity is available for the distribution.

Over the last three decades, many variants to the basic IRP model were introduced. The features that distinguish the model include the types of demand, whether it is treated as deterministic or stochastic and also the planning horizon, be it long-term or short-term. Other features that also considered are the topology, inventory, vehicle fleet and routing [1]. Although the IRP is a long-term problem, most proposed approaches started with a short term version to simplify the solution. It is because the long-term planning problem is already hard to formulate, make it almost impossible to solve [2]. The short-term begins from a single day approach, and later was expanded to several days [3].

Several review papers on IRP are available in the literature. The first review paper was done by [4]. They reviewed a number of available literatures on a class of problems named dynamic routing and inventory (DRAI) problem. Later in 2006, [5] presents an overview of supply chain management related to inventory routing

problem, which highlights the usefulness of the models in practice as well as their limitations. Ref. [1], on the other hand, review around 90 papers on industrial aspects of combined inventory management and routing in maritime and road-based transportation. Meanwhile, [6] gives a comprehensive overview on IRP after thirty years of its establishment. They provide a new classification of the problem and categorized it according to the structural variants and the availability of the information on the customer demand. Table I represent the general classification of IRP introduced by [6].

TABLE I. CLASSIFICATION OF INVENTORY ROUTING PROBLEM

Criteria	Options		
Time Horizon	Finite	Infinite	
Number of item	Single	Multiple	
Structure	One-to-one	One-to-many	Many-to-many
Routing	Direct	Multiple	
Inventory Policy	Maximum level	Order-up-to level	
Inventory Decision	Lost sales	Backlogging	
Fleet Composition	Homogeneous	Heterogeneous	
Fleet Size	Single	Multiple	Unconstrained

Source: Adapted from Coelho *et al.* (2012)

Other elements that need to be highlight in the distribution of item are the delivery mode and the concepts of sharing vehicle. There are 3 types of delivery mode, that is, single delivery, split delivery and multi-drop. In single delivery, a product is delivered using a homogeneous/heterogeneous vehicle to a single customer. Split delivery occurs when a customer's delivery in each period can be split and satisfied by multiple vehicle routes if necessary [7]. Multi drop, on the other hand, refers to demand of a set of customer that is satisfied by only a single vehicle. In some industries, vehicles are shared among different product. For instance, in the distribution of industrial gas, one vehicle can be used to deliver one particular product. Then, once it is emptied and cleaned, another product will be pumped in to the same vehicle.

IRP arises where vendor managed inventory (VMI) is being used to handle complex processes in making sure that customer does not run out of stock. Application of IRP can be seen in many industries, namely, the

distribution of automotive industries, chemical products, frozen product, oil and gas, groceries, ammonia, blood, bitumen and industrial gas [1].

## II. PROBLEM DESCRIPTION

Our study concerns with solving IRP with multi depot, multi product, multi vehicle and heterogeneous vehicle fleet. In Malaysia, the lack of research and development in some industries contribute to logistics and supply chain issue [8]. Even though, both parties, the industries and the academia are aware of the importance of it, the area is still under research [8]. Because of that, and confidentiality matter, real data from the industry are very hard to obtain. Due to that, bench mark instances are one of the option that researcher could have use to validate the model and verify the result obtained. In general, there are two types of data available for analysis purposes, i.e. benchmark instances and real data. Benchmark instances are data that is randomly generated.

In this paper, bench mark instances which were obtained from a website (www.leandro-coelho.com) were used. There are more than 100 sets of 50 customers that are available in this website. Each set of customer are meant for single depot. Since there are lack of available benchmark instances for the problem under consideration (multi depot, multi item with heterogeneous vehicle fleet), we randomly combined 3 set of customers to illustrate 3 depots, to suit our study. The purpose of this study is to obtain a test instances that will be used for validation, once the model has been developed.

## III. GENERAL CLUSTERING METHOD

Clustering is used in a number of traditionally distant fields to describe methods for grouping of unlabelled data. Different research communities have different terminologies for clustering and the context in which cluster techniques are used.

In general, clustering is define as a classification technique to group a set of objects into clusters such that the objects in the same cluster are similar in some sense and those in different clusters are dissimilar in the same sense [9]. Cluster has a wide application in various areas; biology, information retrieval, climate, psychology and medicine as well as business. There are various types of clustering available in the literature. One is hierarchical or nested, partitional or unnested, exclusive, overlapping, fuzzy, complete and partial. Partitional or unnested clustering refers to a group of a set of data objects where each object is in exactly one subset. In hierarchical clustering, subcluster is allowed, where each cluster in the tree is the union of its subcluster and the root of the tree is the cluster containing all of the objects [10].

Proximity or distance measures are often used as the basis for clustering the objects. Several measures have been proposed to determine the proximity between points on the plane, however the most common for quantitative data is the Euclidean metric that determines the proximity between the point  $I = (x_i, y_i)$  and  $J = (x_j, y_j)$  as

$$d = (I, J) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$
. Some measures of proximity among groups have been proposed, which are single linkage, complete linkage, group average, centroid, ward and saving [11].

Clustering can be differentiated based on several types, that are, hierarchical versus. Clustering algorithms may be classified as exclusive clustering, overlapping clustering, hierarchical clustering and probabilistic clustering. Each algorithm has its potential and weaknesses.

### A. Hierarchical Algorithm

In this paper, Hierarchical Algorithm will be implemented to illustrate the problem being study. The following are the procedure for basic Hierarchical Algorithm. Although it has few weaknesses, as a start, the technique could be used for clustering.

Let  $S$  is a set of  $N$  objects,  $S = \{s_1, s_2, \dots, s_N\}$ , to be clustered and a function of distance between 2 clusters  $c_i$  and  $c_j$  as  $D(c_i, c_j)$ .

The basic process of hierarchical clustering is as follows [11],

- 1) Start by assigning each object to a cluster  $c_i = s_i (i = 1, \dots, N)$  so that if there are  $N$  objects, then there are  $N$  clusters  $\ell = \{c_1, c_2, \dots, c_N\}$ , each containing just one item.

Find the pair of clusters  $(c_i, c_j)$  such that  $D(c_i, c_j) \leq D(c_i, c_j) \forall c_i \neq c_j \in \ell$ . Merge them into a single cluster,  $c_k = c_i \cup c_j$ . Cluster  $c_i, c_j$  to be deleted from  $\ell$  and  $c_k$  to be inserted into  $\ell$ .

- 2) The distance similarity between the new cluster and each of the old clusters is to be computed.
- 3) Step 2 and 3 to be repeated until all items are clustered into a single cluster of size  $N$ .

### B. Single Linkage and Complete Linkage Clustering

Step 3 in the hierarchical algorithm can be done in different ways, which are, single linkage, complete linkage and average linkage clustering. In this study, the two clustering techniques will be implemented, to compare the effectiveness [11].

- 1) In a single linkage clustering, the distance between one cluster and another cluster is equal to the shortest distance from any member of one cluster to any member of the other cluster,  $D(c_1, c_2) = \min d(a, b), a \in c_1, b \in c_2$ .
- 2) In complete linkage clustering, the distance between one cluster and another cluster is equal to the greatest distance from any member of one cluster to any member of the other cluster.  $D(c_1, c_2) = \max d(a, b), a \in c_1, b \in c_2$ .

## IV. CONSTRUCTIONS OF TEST INSTANCES

Since there were no available benchmark instances to describe our study, we randomly merged 3 sets of 50 customers out of 45 data sets to illustrate 3 depots. MATLAB 7.12.0 (R2011a) were used. The benchmark instances were obtained from [12]. Each dataset contain the location of the customers and the depot, number of vehicles, number of products and time horizon. Each test instances is created by merging three randomly selected dataset from the 45 sets of data, which makes 15 test instances were created. Later, the new test instances will be used for validation and verification of the developed model, which will be discuss in the upcoming paper.

## V. RESULT AND DISCUSSION

Sample of result are presented. Table II represent the minimum and maximum distance for the three test instances. The points for test instances A are available in the Appendix.

TABLE II. SUMMARY OF THREE TEST INSTANCES

	Test instances A	Test instances B	Test instances C
Min distance	4.000	5.3853	3.1623
Maxi distance	740.8542	671.0623	645.9141

Fig. 1a, 1b and 1c represents the scatter plot for the three instances respectively before clustering is implemented.

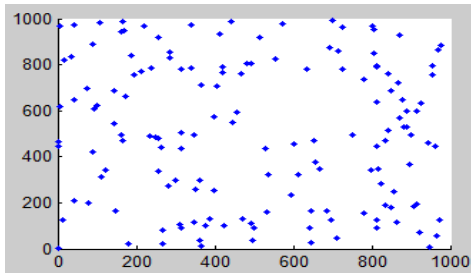


Figure1a Scatter plot of test instances A

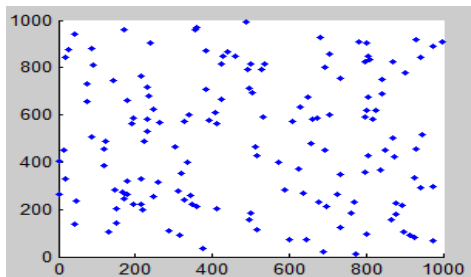


Figure1b Scatter plot of test instances B

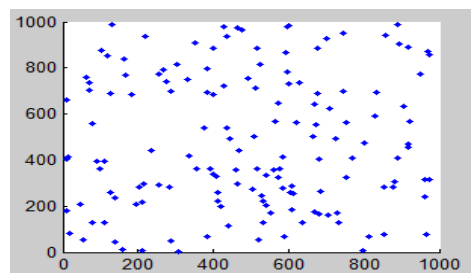


Figure1c Scatter plot of test instances C

Fig. 2a, 2b and 2c depicts the scatter plot for the three instances respectively with single linkage clustering. While Fig. 3a, 3b and 3c depicts the scatter plot using complete linkage clustering. The symbol '\*', 'o' and 'x' represent 3 different clusters.

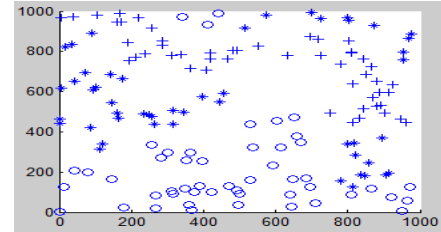


Figure2a Scatter plot of test instances A with single linkage clustering

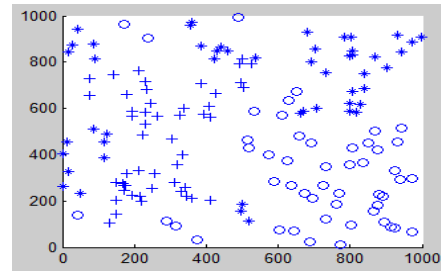


Figure2b Scatter plot of test instances B with single linkage clustering

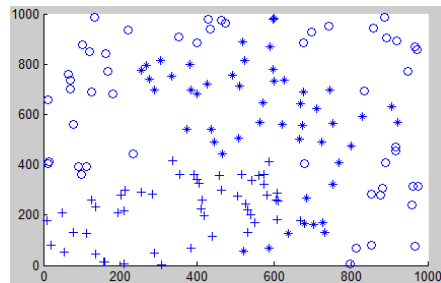


Figure2c Scatter plot of test instances C with single linkage clustering

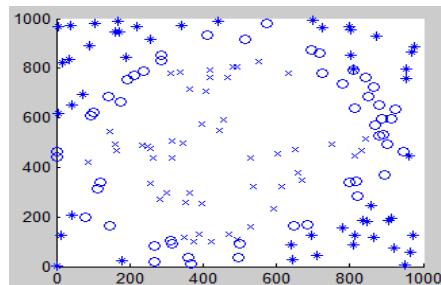


Figure3a Scatter plot of test instances A with complete linkage clustering

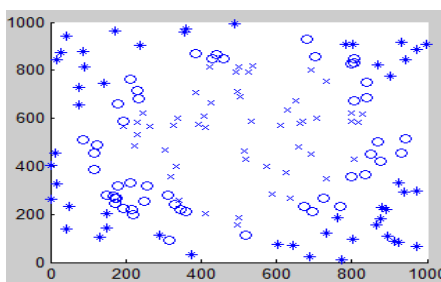


Figure3b Scatter plot of test instances B with complete linkage clustering

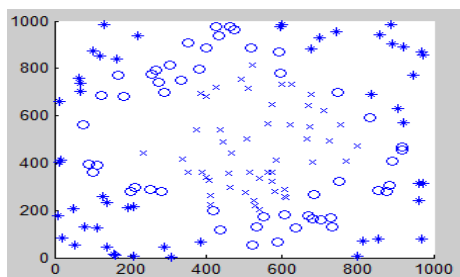


Figure3c Scatter plot of test instances C with complete linkage clustering

Results of the merging of benchmark instances and clustering techniques were presented. Three out of 15 test instances were discussed in this paper. Comparing Fig. 2 and 3, it can be seen that single linkage is quite stable to cluster the data since it uses the minimum distance criteria. Hence, it can be used as an initial solution to further improve the clustering. Other clustering methods could be employed for better results, such as K-Mean. The results obtained in this paper will be used for validation purposes which will be discussed in the near future.

## VI. CONCLUSION

### APPENDIX A POINTS FOR TEST INSTANCES

x	y		x	y		x	y
563	567		157	13		551	171
680	166		505	275		413	261
427	720		541	336		268	793
906	632		333	751		921	570
540	203		963	314		15	414
606	260		77	560		858	942
69	738		597	781		673	555
668	176		160	12		137	234
752	324		522	55		800	474
493	755		195	210		209	5
276	740		437	540		374	540
382	797		597	977		406	328
120	851		163	840		201	281
724	492		677	884		69	702
78	130		111	393		209	216
753	564		561	359		91	394
48	209		385	696		973	857
308	1		354	360		575	363
704	162		892	904		434	938
182	683		686	266		770	408
665	502		444	491		732	130
12	659		611	256		288	47
638	127		137	44		728	170
882	305		919	891		746	698
126	258		917	469		532	129
573	324		102	876		960	241
572	646		289	698		608	288
337	416		676	690		399	341
599	731		12	405		815	69
516	361		967	78		974	314
511	714		831	593		854	284
799	7		607	183		220	936
591	868		876	281		586	412
255	774		283	282		400	886
948	773		889	986		669	643
474	963		523	814		54	53
464	299		621	562		234	442
125	688		418	198		132	986
968	870		834	692		699	929
254	290		401	683		527	243
352	909		508	504		385	68
462	975		531	220		9	179
588	67		458	357		627	735
599	984		918	454		518	887
411	224		100	362		467	442
439	116		710	623		428	978
853	79		583	280		679	403
213	299		304	814		112	128
744	953		168	770		19	82
64	760		391	361		891	408

## REFERENCES

- [1] H. Andersson, A. Hoff, M. Christiansen, G. Hasle, and A. Løkketangen, "Industrial aspects and literature survey: Combined inventory management and routing," *Computers & Operations Research*, vol. 37, no. 9, pp. 1515-1536, 2010.
- [2] A. Campbell, L. Clarke, A. Kleywegt, and M. W. P. Savelsbergh, "The inventory routing problem," *Fleet Management and Logistics*, pp. 95-113, 1998.
- [3] A. Campbell, L. Clarke, and M. W. P. Savelsbergh, "Inventory routing in practice," *The Vehicle Routing Problem*, vol. 9, pp. 309-330, 2002.
- [4] F. Baita, W. Ukovich, R. Pesenti, and D. Favaretto, "Dynamic routing-and-inventory problems: A review," *Transportation Research Part A: Policy and Practice*, vol. 32, no. 8, pp. 585-598, 1998.
- [5] N. H. Moin and S. Salhi, "Inventory routing problems: A logistical overview," *Journal of the Operational Research Society*, vol. 58, no. 9, pp. 1185-1194, 2006.
- [6] L. C. Coelho, J. F. Cordeau, and G. Laporte, "Thirty years of inventory-routing," *Transportation Science*, vol. 48, no. 1, pp. 1-19, Feb. 2014.
- [7] Y. Yu, C. Chu, H. Chen, and F. Chu, "Large scale stochastic inventory routing problems with split delivery and service level constraints," *Annals of Operations Research*, vol. 197, no. 1, p. 135, 2012.
- [8] R. Ali, H. S. Jaafar, and S. Mohamad, "Logistics and supply chain in malaysia : Issues and challenges," in *Proc. EASTS International Symposium on Sustainable Transportation Incorporating Malaysian Universities Transport Research Forum Conference*, 2008, pp. 1-11.
- [9] M. R. Anderberg, *Cluster Analysis for Application*, Academic Press, 1973.
- [10] S. Barreto, C. Ferreira, J. Paixão, and B. S. Santos, "Using clustering analysis in a capacitated location-routing problem," *European Journal of Operational Research*, vol. 179, no. 3, pp. 968-977, Jun. 2007.
- [11] L. P. K. Agarwal, *Lecture 18 : Clustering & Classification*, 2003, pp. 1-9.
- [12] L. C. Coelho and G. Laporte, "A branch-and-cut algorithm for the multi-product multi-vehicle inventory-routing problem," *International Journal of Production Research*, vol. 51, pp. 7156-7169, 2012.



**Norlenda Mohd Noor** received her BSc (Mathematics) and MSc (Mathematics) from the Universiti Kebangsaan Malaysia in 2000 and 2001, respectively. Currently, she is a PhD student in Mathematics at the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Malaysia. Her research interest includes optimization problem, mathematical modelling, and heuristics.



**Adibah Shuib** received her BSc (Math) and MSc (Computational & App Math) degree from the Old Dominion University Norfolk, Virginia, USA, in 1987 and 1988, respectively. In 2007, she obtained her PhD (Management Mathematics) degree from University of Birmingham, UK, 2007. Currently, she is an Associate Professor at the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Malaysia since 2001. Her research interests

are mathematical modeling, specifically for solving optimization problems. These include Exact and Metaheuristics Approaches for Vehicle Routing problem, Diet Planning, Scheduling, including Portfolio and Budget Optimization.