Optimization of the Hub Height of a Wind Turbine

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***Abstract*—This paper presents a new method for optimizing the hub height of a wind turbine. In general, wind turbine energy production increases with the hub height, but so does the cost. Therefore, we must optimize the hub height. Here, we calculated the annual energy production using a wind probability function. This is a function of the hub height and the roughness, which is a surface characteristic. The wind turbine cost was also expressed as a function of the hub height. The objective function for the optimization process was formulated in terms of the annual energy production and wind turbine cost. Differentiation was used to carry out the optimization; the procedure is described in this paper. Finally, the results of a case study were used to illustrate the relationship between the optimum hub height and the roughness.**

***Index Terms*—wind turbine hub height, roughness, optimization, rayleigh probability density function**

I. Introduction

Wind energy is pollution-free and renewable, uses no fossil fuels, and thus can provide an effective remedy for fossil-fuel depletion. For this reason, the use of wind energy has continued to expand, and will play an important role in the generation of electricity in the future. According to the Global Wind Energy Council (GWEC) [1], the cumulative global wind-power capacity was about 10 GW in 1998, and had increased to about 237 GW by 2011. The amount of research devoted to wind turbines has also been increasing worldwide.

Optimization of capacity is an important consideration in designing any turbine, including a wind turbine. Optimum capacity of wind turbines and wind turbine systems combined with energy-storage systems has already been studied by numerous researchers [2]–[4]. For a wind turbine, however, we must consider not only the capacity, but also the hub height. We can cut the cost by optimizing the hub height, since the cost of the tower comprises about 15 – 25% of the initial capital cost (*ICC*) of a wind turbine [5] [6]. The amount of existing research on optimum hub height is relatively small in comparison with the amount of research on optimum capacity. Accordingly, we investigated a technique for optimizing the hub height.

In this paper, we propose a method that uses differentiation to optimize the hub height. We used the method to analyze the relationship between the optimum hub height and the roughness determined by the surface characteristics, and validated the proposed method by comparison with the results of another study.

II. Background

We optimized the hub height, h for a single wind turbine. The following figures are provided to better explain our research topic.

Wind speed increases with height, as shown in Fig. 1. In general, energy production increases as h increases due to the increased wind speed. However, the tower cost also increases in proportion to h, and hence we must determine the value of h that maximizes the profit from the wind turbine.

Fig. 2 shows the effect of h on the annual energy production (*AEP*) of a wind turbine. The wind probability function varies with h, but the power curve of the wind turbine remains unchanged. Since *AEP* is calculated using the wind probability function and the power curve of the wind turbine, it can be determined from the variable h.

We made the following assumptions to solve this problem:

- All electricity is sold to the grid.
- The cost function is based on a tubular steel tower.
- The operating and maintenance cost is linearly proportional to *AEP*.

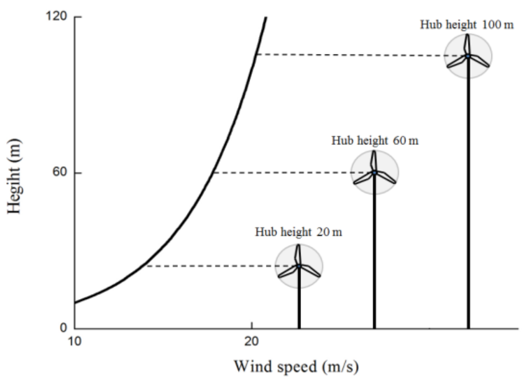

Figure 1. Wind speed versus hub height

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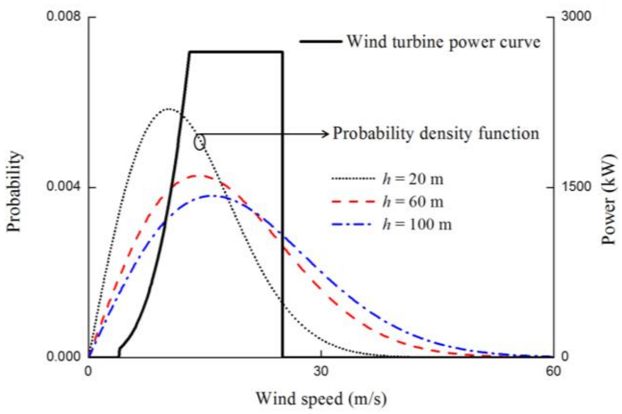

Figure 2. Wind turbine power curve & wind probability density function

We formulated the objective function *Obj* based on the above assumptions. *Obj* is the annual net profit, taking *AEP* and cost into account. The profit from selling electricity is expressed as the product of *AEP* and the cost of electricity, C_e. The annual operating and maintenance costs are obtained by multiplying *AEP* by the operating and maintenance costs per kWh, C_{AOM}.

ICC can be converted to an annual cost by dividing by the lifespan *N* of the wind turbine. *Obj* is then given by the following equation:

$$Obj = \mathrm{C_e} AEP - \mathrm{C_{AOM}} AEP - ICC / N \qquad (1)$$

III. Mathematical Modeling

A. Wind-Speed Distribution

The Rayleigh distribution was used to model the wind distribution in [7]. This is a special case of a Weibull distribution, determined by a scale parameter and a shape parameter, in which the shape parameter is equal to 2 and given by

$$pdf\,(v) = \frac{\pi}{2}\frac{v}{\bar{v}^2} e^{-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2} \qquad (2)$$

TABLE I. Surfaces Description and the Roughness Values

Description	Roughness value range (m)	Most likely value (m)
Continuous urban fabric	1.1 - 1.3	1.2
Broad-leaved forest; Coniferous forest; Mixed forest	0.6 - 1.2	0.75
Green urban areas; Transitional woodland/shrub; Burnt areas	0.5 - 0.6	0.6
Discontinuous urban fabric; Construction sites; Industrial or commercial units; Sport and leisure facilities; Port areas	0.3 - 0.5	0.5
Agro-forestry areas; Complex cultivation patterns; Land principally occupied by agriculture; with significant areas of natural vegetation	0.1 - 0.5	0.3
Annual crops associated with permanent crops; Fruit trees and berry plantations; Vineyard; Olive groves	0.1 - 0.3	0.1
Road and rail networks and associated land	0.05 - 0.1	0.075
Non-irrigated arable land; Permanently irrigated land; Rice fields; Salt marshes		0.05
Sclerophylous vegetation; Moors and heathland; Natural grassland; Pastures	0.03 - 0.1	0.03
Dump sites; Mineral extraction sites; Airports; Bare rock; Sparsely vegetated areas		0.005
Glaciers and perpetual snow		0.001
Peat bog; Salines; Intertidal flats		0.0005
Water courses; Water bodies; Coastal lagoons; Estuaries; Sea and ocean		0

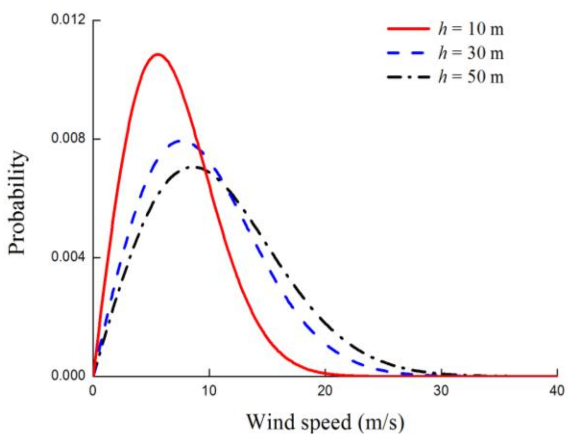

Figure 3. Probability density function for various hub height

where $pdf(v)$ denotes the wind probability function and $\bar{v}$ is the average wind speed.

The logarithmic law [8] can be used to write $\bar{v}$ as follows:

$$\bar{v}(h) = \bar{v}(h_{\mathrm{ref}}) \frac{\ln\left(\frac{h}{z_0}\right)}{\ln\left(\frac{h_{\mathrm{ref}}}{z_0}\right)} \qquad (3)$$

where h_{ref} is the reference height at which the wind speed is measured and z_0 is the roughness. Thus, $pdf(v)$ varies according to h. Fig. 3 shows $pdf(v)$ for various h when z_0, $\bar{v}$, and h_{ref} are 0.5, 7 m/s and 10 m, respectively. Here, z_0 is a factor affecting the characteristics of the $pdf(v)$ variation; Table I lists its values for various surfaces [9]. Note that $pdf(v)$ shifts to the right as h increases.

B. Wind Turbine Power Curve

The power output, $P(v)$ of a wind turbine is calculated via the following equation:

$$P(v)=\frac{1}{2}\rho\pi R^2 C_p v^3 \tag{4}$$

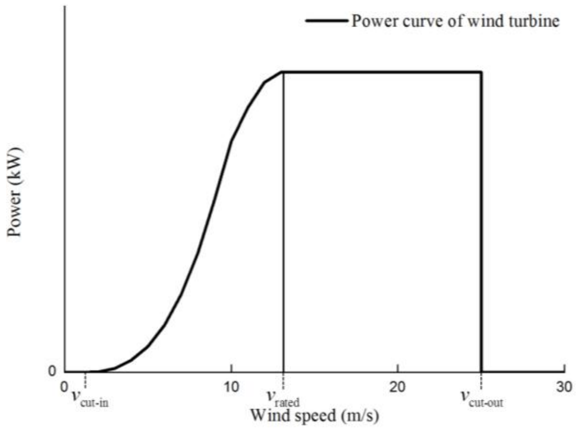

Figure 4. Typical Power curve for a wind turbine

Hence, $P(v)$ is proportional to the cube of the wind speed v and the square of the rotor radius R. Here, ρ denotes the air density and C_p is the power coefficient.

Fig. 4 shows a typical power curve for a wind turbine. The power curve is divided into three parts in accordance with the following equation:

$$P(v)=\begin{cases} 0 & v<v_{\text{cut-in}} \text{ or } v>v_{\text{cut-out}} \\ \frac{1}{2}\rho\pi R^2 C_p v^3 & v_{\text{cut-in}}<v<v_{\text{rated}} \\ P_{\text{rated}} & v_{\text{rated}}<v<v_{\text{cut-out}} \end{cases} \tag{5}$$

Here, $v_{\text{cut-in}}$ is the cut-in speed (the minimum speed for wind generation), v_{rated} is the rated speed (at which $P(v)$ attains the rated power output P_{rated}), and $v_{\text{cut-out}}$ is the cut-out speed (the maximum speed, used to protect the turbine from winds that are too strong).

C_p varies with the wind speed. Each wind turbine has unique value of C_p. We assumed that C_p is constant and has a maximum value of $C_{p,max}$, as provided by the turbine manufacturer. C. Carrillo, A. F. Obando Montano, J. Cidras, E. Diaz-Dorado [10] demonstrated that this method is effective for modeling C_p. Thus, P_{rated} is given by

$$P_{\text{rated}}=\frac{1}{2}\rho\pi R^2 C_{p,\max} v_{\text{rated}}^3 \tag{6}$$

C. Annual Energy Production

AEP is calculated from the following equation:

$$AEP=8760\times\int_{v_{\text{cut-in}}}^{v_{\text{cut-out}}} P(v)\,pdf(v)\,dv \tag{7}$$

The integral term in (7) is the expected wind turbine output (kW).

D. Initial Capital Cost

ICC can be divided into two parts: the tower cost ICC_{tower} and the total cost excluding the tower, ICC_{WTG}. Hence

$$ICC=ICC_{\text{WTG}}+ICC_{\text{tower}} \tag{8}$$

ICC_{tower} is the product of the tower mass M_{tower} and the cost per kg of the steel, C_{steel}. According to L. Fingersh, M. Hand, A. Laxson [11], ICC_{tower} and M_{tower} can be written as:

$$\begin{aligned} ICC_{\text{tower}} &= M_{\text{tower}} C_{\text{steel}} \\ M_{\text{tower}} &= 0.2694Ah+1779 \end{aligned} \tag{9}$$

ICC_{WTG} is expressed as follows [5]:

$$ICC_{\text{WTG}}=P_{\text{rated}}C_{\text{WTG}} \tag{10}$$

ICC_{WTG} is proportional to P_{rated}, and is the total cost of the wind turbine excluding the tower (including the rotor, hub, gear, etc.).

IV. Optimum Hub Height

A. Formulation of the Optimization Problem

We can calculate *Obj* by substituting the results obtained from the mathematical model. At the optimum hub height h_{opt}, *Obj* is maximized. We formulated the optimization problem as follows:

$$\begin{aligned} &\text{Find } h_{\text{opt}} \\ &\text{Maximize } Obj \\ &\text{Subject to } 1.5R<h<3R \end{aligned} \tag{11}$$

Here, h is restricted by R. If h is too large in comparison to R, the economics are adversely affected, and it is physically impossible for h to be smaller than R. Thus, we need to impose some constraints on h. We used the constraint $1.5R < h < 3R$, which was employed by Mohammad Rezaei Mirghaed, Ramin Roshandel [12].

B. Optimization Process

Our problem is a constrained optimization problem, and *Obj* is continuous and differentiable for all positive h. The method for finding h_{opt} via differentiation is introduced in this section. First, *Obj* is differentiated with respect to h as follows:

$$\begin{aligned} \frac{d}{dh}Obj &= (C_e - C_{AOM})\frac{d}{dh}AEP - \frac{1}{N}\frac{d}{dh}ICC \\ &= 8760(C_e - C_{AOM})\int_{v_{\text{cut-in}}}^{v_{\text{cut-out}}} P(v)\frac{d}{dh}pdf(v)\,dv \\ &\quad -\frac{1}{N}\frac{d}{dh}ICC \end{aligned} \tag{12}$$

The first term of (12) involves the derivative of the net profit, and the second term involves the derivative of the cost. *Obj* attains an extreme value when (12) equals 0. The derivative of the cost is a positive constant, since *ICC* is a first-order function of h. If h exceeds a specific value, the derivative of *AEP* is negative due to the increased probability that v is greater than $v_{\text{cut-out}}$. Thus, the derivative of *AEP* is positive for small h, and becomes negative beyond a certain point. However, *AEP* cannot be negative, even when h is very large. Therefore, the derivative of *AEP* converges to 0 from that point.

Based on these observations about the derivatives of *AEP* and *ICC*, the expression in Eq. (12) is positive for small *h* and becomes negative beyond a specific point. When (12) equals 0, *Obj* attains a maximum value.

TABLE II. THE VALUES OF THE PARAMETERS IN THE CASE STUDY

Parameters	values
Rotor radius (m)	40
$C_{p,\,max}$	0.4
air density (kg/m^3)	1.22
cut-in speed (m/s)	4
rated speed (m/s)	13
cut-out speed (m/s)	25
C_e (\$/kWh)	0.15
C_{AOM} (\$/kWh)	0.02
h_{ref} (m)	10
C_{steel} (\$/kg)	2.4
N (year)	20

V. CASE STUDY

In this section, a case study is investigated via the proposed method.

A. Conditions for the Case Study

The values of the parameters used in the case study are listed in Table II. C_{AOM} [13] is 0.02 \$/kWh and *N* [14] is 20 years. z_0 = 1, 0.5, 0.1, and 0.05, and $\bar{v}$ = 7 – 11 m/s.

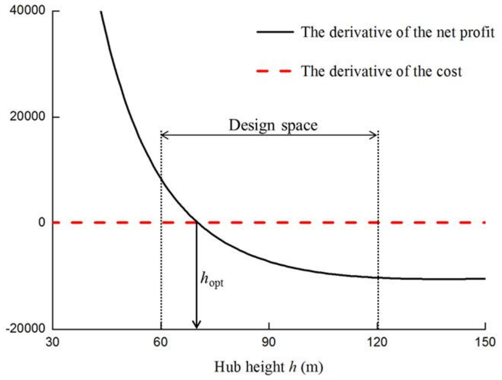

Figure 5. The graphs of the derivatives with hub height

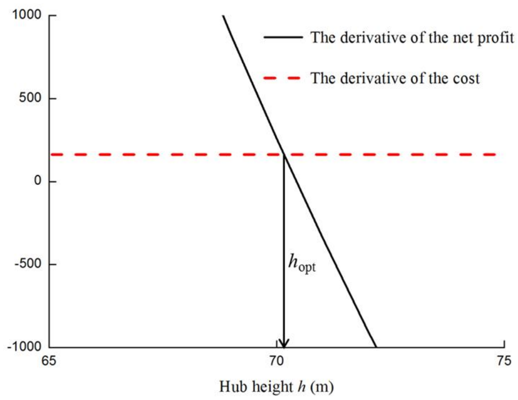

Figure 6. The graphs of the derivatives near the optimum hub height

TABLE III. THE RESULT OF THE CASE STUDY

Z_0	1	0.5	0.1	0.05
$\bar{v}$	h_{opt} (m)	h_{opt} (m)	h_{opt} (m)	h_{opt} (m)
7	120	120	120	120
7.5	93	120	120	120
8	70.2	120	120	120
8.5	60	90.8	120	120
9	60	68.2	120	120
9.5	60	60	120	120
10	60	60	89.3	120
10.5	60	60	64.9	85.5
11	60	60	60	61.2

B. Results

Fig. 5 and Fig. 6 show the results of the case study for z_0 = 1 and $\bar{v}$ = 8 m/s. For small *h*, the derivative of the profit term was greater than that of the cost term. The two graphs intersected at h_{opt}. We inferred that the derivative of the net profit became negative, which means that *AEP* decreased, since *pdf*(*v*) is not suitable for the power curve of a wind turbine when *h* is too large.

The results of the case study are presented in Table III. When z_0 was low, h_{opt} remained at 120 m over a wide range of $\bar{v}$, because the wind speed was insufficient for the wind turbine. The higher the value of z_0, the lower h_{opt} became for a given wind speed.

Thus, sufficient wind could be obtained at a low *h* if z_0 was high. The same trend was observed by M. H. Albadi, E. F. El-Saadany [15].

VI. CONCLUSION

We proposed a method for optimizing the hub height of a specific wind turbine. To optimize the hub height, we expressed the cost and energy production as functions of the hub height, and formulated an objective function that represented the annual net profit from the wind turbine. This objective function could be differentiated with respect to the hub height to find the optimum hub height. When the derivative of the objective function was equal to 0, the net profit was maximized, and the optimum hub height could be determined. We also inferred that energy production might decrease when the hub height is too large. This accentuates the necessity for research on hub height.

The wind probability density function is determined from the hub height and roughness. The proposed method also requires the power curve of the wind turbine. Thus, the optimization process requires only the roughness of a specific surface and the power curve of the wind turbine. A simple power curve model was used in this paper, but the method can be applied to any wind turbine.

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