Calculation of Hole Collapse Pressure Considering the Influence of Borehole Diameter

Jin Sun and Jingen Deng
Department of Petroleum Engineering, China University of Petroleum(Beijing), Beijing, China
Email: sunjin19870216@126.com, dengjg@cup.edu.cn

Abstract—To solve the wellbore instability problem of slimhole, this paper presents a calculation method of collapse pressure base on the scale effect theory of rock strength. Based on the empirical relation between the uniaxial compressive strength and the specimen diameter proposed by Hoek, a modified empirical relation is studied in which the size effect of internal frictional angle is considered. A modified Mohr-Coulomb failure criterion with the influence of scale effect of rock strength is established. The collapse pressure which considers the influence of borehole diameter is calculated, and two evaluation method of size effect of rock strength is presented. The results show that the collapse pressure of slimhole is related to the borehole diameter. Meanwhile, the scale effect of formation with different properties is also different. The reduction in strength is due to the nonhomogeneity and fracture development of rocks. This provides important reference for the evaluation of collapse risks of wells with different diameters.

Index terms—slimhole, borehole diameter, scale effect, collapse pressure, fracture development

I. INTRODUCTION

There are many marginal oilfields in China offshore oilfield and how to find a more economic and effective way to development the offshore marginal oilfield is very important for improving oil production and reducing China’s dependence on foreign oil [1]. According to marginal oilfields development experience of some international oil corporations, slimhole drilling and monobore completion technique is an effective way for the development of offshore marginal oil field. For example, Unocal corporation put this technique into practice in Gulf of Tailand, and this greatly simplifies the well configuration [2] [3]; Malaysia’s Petronas reduce the drilling and completion costs by using monobore completion in East Malaysia [4]. Other corporations (such as Chevron [5]) have piloted the similar technique too. However, at present this technology has not been put into practice in offshore oilfields in China. Because this technique has been successfully used in some offshore oilfields, it’s essential to evaluate the suitability of slimhole drilling and monobore completion technique in China offshore oilfields.

Borehole instabilities is often characterized by the slabling mode that affects a portion of the material close to the borehole wall. In addition, it is clear that the borehole size has significant effect on the hole collapse. Slimhole has a smaller hole diameter, and compared with conventional wells, it is more difficult to deal with drilling accidents. During the process of slimhole drilling, we should put more attention to the well instability problems, especially hole collapse. This paper give a new calculation method to predict the collapse pressure of slimhole while drilling, and example calculation is done by using this method. The result shows that the size effect of rock strength is the main difference between slimhole and conventional well.

II. THE DIFFERENCE OF WELL STABILITY BETWEEN SLIMHOLE AND CONVENTIONAL WELLS

Rock is a type of natural material, and there are some internal defects in rock, such as cracks, pores, joints [6]. These characteristics lead to the nonhomogeneity of rock properties. And some rocks which have poor qualities may exist some macro discontinuities (macro cracks, faults, et al.), this exacerbate the size effect of rocks.

At present, the research method of size effect of rock strength contains theoretical analysis, numerical simulation and experimental methods. For theoretical analysis, researchers mainly study the size effect from the microstructure of rocks. For numerical simulation, the major method is discrete element method. For instance, UDEC and PFC are usually used to simulate the defects in the rocks [7] [8]. For experimental methods, the relation of stain and stress and strength is studied, based on the experimental result, some empirical relations between the sample size and the strength are built. Because the experimental method is relatively simple, and the mechanical parameters are easy to obtain from lab and logging information, it becomes the main way to study the size effect of rocks.

The relations between uniaxial compressive strength (UCS) and rock sample size are mostly used to describe the size effect of rocks. For example, Kostak and Bienlein established an empirical relations between UCS and rock sample volume [9]; Liu indicates that UCS shows exponential decline with rock sample diameter [10]; Hoek has studied the size effect with different types and size of rocks [11] [12], the experimental results are
shown in Fig. 1. It is concluded that UCS is getting lower with the increase of sample diameter.

A power law relationship between UCS and rock sample size is put forward by Hoek to characterize the strength size effect of rocks, it is given by

\[ \frac{UCS}{UCS_{50}} = \left( \frac{D}{50} \right)^n \]  

where \( UCS \) is uniaxial compressive strength, \( UCS_{50} \) is uniaxial compressive strength of cylindrical specimen with diameter \( D=50\)mm, \( D \) is diameter of specimen, \( n \) is a positive empirically derived constant , it depends on the type of rocks. Howerver , it should be noticed that this relationship is only limited to intact rocks.

\[ \sigma_v = -p + (\sigma_H + \sigma_r) - 2(\sigma_H - \sigma_z) \cos 2\theta + \alpha \left( \frac{\alpha(1-2\nu)}{1-\nu} - \phi \right) (P - P_p) \]  

(2)

\[ \sigma_r = P - \delta \phi (P - P_p) \]  

(3)

\[ \sigma_z = \sigma_r - (2(\sigma_H - \sigma_z) \cos 2\theta) + \alpha \left( \frac{\alpha(1-2\nu)}{1-\nu} - \phi \right) (P - P_p) \]  

(4)

where \( \sigma_t \) is the tangential stress, \( \sigma_r \) is the radial stress, \( \sigma_z \) is the vertical stress. \( \theta \) is the angle between the point on the borehole wall and the maximum horizontal stress direction. \( \delta \) is the permeability coefficient, when the borehole wall is permeable, \( \delta =1 \); and else , \( \delta =0 \). \( p \) is the internal pressure. \( p_p \) is the pore pressure. \( \nu \) is poisson’s ratio. \( \phi \) is the porosity. \( \alpha \) is the Biot coefficient.

It can be concluded that the specimen size will affect greatly on the strength of rocks, the smaller the specimen is, the stronger the strength of the specimen is. And it indicates that the diameter of wells will influence the well instability during drilling. For different diameters, the collapse pressure will be different too. Our goal is to find a calculation method to evaluate the size effect of collapse pressure during drilling in hard rock formation.

### III. CALCULATION OF COLLAPSE PRESSURE CONSIDERING THE INFLUENCE OF BOREHOLE DIAMETER

By using linear elastic theory, the stress state around well is obtained, and based on the empirical relation between the uniaxial compressive strength and the specimen diameter proposed by Hoek, A modified Mohr-Coulomb failure criterion with the influence of scale effect of rock strength is established . Then we substitute the principle effective stresses on the borehole wall into the modified Mohr-Coulomb failure criterion to calculate the collapse pressure.

#### A. The Calculation Model of Hole Collapse Pressure

When a well is drilled into a formation, stressed solid material is removed. There will be stress redistribution around the well. We assume that there is a borehole in infinite formation, and it is loaded with an internal pressure \( p \), and in infinity, it is loaded with the maximum minimum horizontal stress \( \sigma_H \) and the minimum horizontal stress \( \sigma_h \). Here we use the plane strain model to get the stresses around the well, the mechanical model is shown in Fig. 2.

According to the linear elastic theory and superposition principle, the stresses on the borehole wall are obtained:

\[ \sigma_v = -p + (\sigma_H + \sigma_r) - 2(\sigma_H - \sigma_z) \cos 2\theta + \alpha \left( \frac{\alpha(1-2\nu)}{1-\nu} - \phi \right) (P - P_p) \]  

(2)

\[ \sigma_r = P - \delta \phi (P - P_p) \]  

(3)

\[ \sigma_z = \sigma_r - (2(\sigma_H - \sigma_z) \cos 2\theta) + \alpha \left( \frac{\alpha(1-2\nu)}{1-\nu} - \phi \right) (P - P_p) \]  

(4)

where \( \sigma_t \) is the tangential stress, \( \sigma_r \) is the radial stress, \( \sigma_z \) is the vertical stress. \( \theta \) is the angle between the point on the borehole wall and the maximum horizontal stress direction. \( \delta \) is the permeability coefficient, when the borehole wall is permeable, \( \delta =1 \); and else , \( \delta =0 \). \( p \) is the internal pressure. \( p_p \) is the pore pressure. \( \nu \) is poisson’s ratio. \( \phi \) is the porosity. \( \alpha \) is the Biot coefficient.

Hole collapse usually occurs at the borehole wall along the minimum horizontal stress direction, so that means that \( \theta = \pi /2 \text{ or } 3\pi /2 \). We can write the effective stress as follows:

\[ \sigma_v = 3\sigma_H - \sigma_z - P + \delta \left( \frac{\alpha(1-2\nu)}{1-\nu} - \phi \right) \times (P - P_p) - \alpha P \]  

(5)

\[ \sigma_r = P - \delta \phi (P - P_p) - \alpha P \]  

(6)

\[ \sigma_z = \sigma_r + 2(\sigma_H - \sigma_z) \cos 2\theta + \delta \left( \frac{\alpha(1-2\nu)}{1-\nu} - \phi \right) (P - P_p) \]  

(7)

Shear failure will occur when the mud density is too small during drilling. Here we use Mohr-Coulomb criterion to describe the shear failure:
\[ \sigma_i = \sigma_i K^2 + 2CK \]  
\[(8)\]

where \( K = \tan(45^\circ + \varphi /2) \), \( \varphi \) is the internal frictional angle, and \( C \) is the cohesion of the rock.

According to Mohr-Coulomb criterion, uniaxial compressive strength (UCS) is expressed as:

\[ UCS = 2CK \]  
\[(9)\]

From equation (9), UCS depends both on the internal frictional angle \( \varphi \) and cohesion \( C \). From the previous analysis, we know that UCS is a function of rock size. So \( \varphi \) and \( C \) are also the function of rock size. The modified Mohr-Coulomb criterion is defined as:

\[ \sigma_i = \sigma_i K^2(D) + UCS_{\text{eq}}(D)z^n \]  
\[(10)\]

where \( D \) is the specimen diameter.

When \( \sigma_{i0} > \sigma_{i} > \sigma_{i}' \), the maximum effective stress is \( \sigma_{i0} \), and the minimum effective stress is \( \sigma_{i}' \), We substitute \( \sigma_{i0} \) and \( \sigma_{i}' \) into equation (10), the equivalent mud density of collapse pressure can be expressed as:

\[ \rho_i = \frac{n(\sigma_{i0} - \sigma_{i} - \Delta \rho_p) - K'(D)(\Delta \varphi - \alpha)\rho_p - \rho_{i0} - UCS_{\text{eq}}(D)z^n}{[K'(D) - \Delta \varphi + n(1-A)]gH} \]  
\[(11)\]

where \( \rho_i \) is the equivalent mud density of collapse pressure, \( A = \delta[\alpha(1-2\nu)/(1-\nu) - \varphi] \), \( \eta \) is non-linear correction coefficient \( (\eta < 1) \).

When the quality of filter cake is very good, the permeability can be negligible. Then the equivalent mud density of collapse pressure can be expressed as:

\[ \rho_i = \frac{n(3\sigma_{i0} - \sigma_{i} - \Delta \rho_p) - K'(D)(\Delta \varphi - \alpha)\rho_p - \rho_{i0} - UCS_{\text{eq}}(D)z^n}{[K'(D) + \eta]gH} \]  
\[(12)\]

when \( \sigma_{i0} > \sigma_{i} > \sigma_{i}' \), the maximum effective stress is \( \sigma_{i0} \), and the minimum effective stress is \( \sigma_{i}' \). Then we substitute \( \sigma_{i0} \) and \( \sigma_{i}' \) into equation (10), the equivalent mud density of collapse pressure with consideration of influence of hole diameter and non-linear can be expressed as:

\[ \rho_i = \frac{n(\sigma_{i0} + 2(\sigma_{i0} - \sigma_{i} - \Delta \rho_p) - K'(D)(\Delta \varphi - \alpha)\rho_p - \rho_{i0} - UCS_{\text{eq}}(D)z^n)}{[K'(D) + \Delta \varphi - A\varphi]gH} \]  
\[(13)\]

When the permeability can be negligible, the equivalent mud density of collapse pressure can be expressed as:

\[ \rho_i = \frac{n(\sigma_{i0} + 2(\sigma_{i0} - \sigma_{i}) - K'(D)(\Delta \varphi - \alpha)\rho_p - \rho_{i0} - UCS_{\text{eq}}(D)z^n)}{K'(D)gH} \]  
\[(14)\]

From equation (11-14), it can be concluded that \( \rho_i \) is related to hole diameter \( D \), the size effect of \( \rho_i \) depends on the size effect of UCS, \( \varphi \) and \( C \).

It should be noticed that equation (11-14) is only applicable for vertical wells. For deviated wells, the stresses around the well will change. In view of this situation, the stresses around the well can be calculated by the deviation angle, the well azimuth and In-situ stresses, then the principle effective stresses is obtained, and \( \rho_i \) can be can be obtained by substituting the principle effective stresses into the modified Mohr-Coulomb criterion (equation 10).

### B. Calculating Example

To study the influence of hole diameter of collapse pressure, the calculation examples are done by using the parameters in Table 1.

The influence of \( n \) on the collapse pressure

Different types of rock have different levels of size effect. Here we assume that the size effect of the internal frictional angle can be negligible, this means that the size effect of UCS depends only on cohesion \( C \). 4 different values of \( n \) is used to evaluate the influence of \( n \) on the collapse pressure, the results are shown in Fig. 3.

From Fig. 3 we can conclude that the collapse pressure is related to hole diameter. If \( n = 0 \), this indicates that there is no size effect, and the collapse will keep constant. And when \( n \neq 0 \), the collapse will change. The smaller the hole diameter is, the lower the collapse pressure is, and this suggest that with the increase of hole diameter, the well is more and more unstable. We can also conclude that with the increase of \( n \), the collapse pressure is getting higher (for example, when hole diameter is 300mm, the equivalent mud density of collapse pressure changes from 0.95 to 1.18). That means that when the size effect is getting stronger, the difference between collapse pressure of well with different hole diameter is getting greater.

The influence of \( K \) on the collapse pressure

From equation (11-14), the collapse pressure is related to \( K \), and \( K \) is a function of the internal frictional angle. Equation (9) shows that UCS is related to \( K \), so \( K \) is a function of size too. Here we assume that there is a power law relation between UCS and rock sample size.

### TABLE I. THE CALCULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Number of data</th>
<th>Parameter name</th>
<th>Number of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth(m)</td>
<td>1000</td>
<td>Pore pressure(MPa)</td>
<td>10</td>
</tr>
<tr>
<td>Biot coefficient</td>
<td>0.95</td>
<td>( \eta )</td>
<td>0.95</td>
</tr>
<tr>
<td>Maximum horizontal stress(MPa)</td>
<td>23.78</td>
<td>Minimum horizontal stress(MPa)</td>
<td>15.57</td>
</tr>
<tr>
<td>UCS of D50mm specimen(MPa)</td>
<td>35</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
The influence of the size effect of collapse pressure with the influence of different hole diameter is studies (n=0.1). The results are shown in Fig. 4.

From Fig. 4 we can conclude that the collapse pressure is related to hole diameter and K. When K remains the same, the size effect of collapse pressure only depends on the size effect of cohesion, and the collapse pressure will increase with the increase of hole diameter. When K increases with size, the internal frictional angle increases too. This indicates that the size effect of collapse pressure is weakened. And When K decreases with size, the internal frictional angle decreases too. This indicates that the size effect of collapse pressure is strengthened.

It is concluded that hole collapse pressure is related to hole diameter on account of the size effect of rock strength. In fact, the size effect of hole collapse pressure is a result of cohesion combined with the internal frictional angle.

IV. THE EVALUATION METHOD OF SIZE EFFECT OF ROCK STRENGTH

The size effect of rock strength is related closely to the development of pore and fractures. With the increase of nonhomogeneity, the size effect will get stronger. Here we can use experimental method and logging information to evaluate the size effect of rock strength.

A. Experimental Method

During the process of drilling, the rock cores can be obtained in different formation. And the rock uniaxial compression experiments and triaxial compression experiments on MTS can be conducted by using the rock cores with different sizes from the same formation, then the UCS, the internal frictional angles and cohesion will be calculated. Finally the relation between the rock strength parameters and specimen size can be established.

B. Logging Information

Because the number of cores is usually very small, it is necessary to use logging information to evaluate the size effect of rock strength. Since the size effect of rock strength depends on the fractures in the rock, we need to find how to describe the fracture with logging information. The logging methods which is sensitive to fracture include resistivity logging, acoustic logging, neutron logging, density logging, compensated density logging, electromagnetic propagation logging, imaging logging.

Here we use fracture index m to describe the fracture development level. A large m means that there are many fractures in the rock. Because the responses of different logging methods is very different, we use fuzzy mathematic to establish the relation between fracture index m and logging parameters. The logging data are normalized as follows:

\[ z_i = (x_i - a)/(b - a) \]  

where \( z_i \) is the normalized logging data, \( x_i \) is the logging data, a and b is constant, usually represented as the maximum and minimum of the data.

A membership function as follow is used to describe fracture index \( m_i \) for each logging method:

\[ m_i = \frac{1}{1 + \exp[-c(z_i - d)]} \]  

where \( m_i \) is the fracture index for specified logging method. c and d is constant, which depends on the relationship between fracture index and logging data.

The fracture index m can be expressed by:

\[ m = \sum w_i \times m_i \]  

where \( w_i \) is weighting coefficient.

By using imaging logging, the fractures can be seen from the image. We will use the imaging logging to verify this method. Fig. 5 shows 3 images around the well in 3 different sections. It can been seen that there are many fractures in the sections which depth is between 3762-3764m, 3958-3960m and 4022-4024m. Fig. 6 shows the vertical distribution of fracture index by using acoustic logging, resistivity logging, GR logging, SP logging. The result indicates that m is large in Huagang formation and Pinghu formation, this is consistent with the imaging logging result.

The larger m is, the higher the fracture development level is, and this leads to strong size effect. So it is important to study the relationship between logging data and the fracture development level for evaluating the size effect of rock strength. This still needs further research.
V. CONCLUSION

Based on the empirical relation between the uniaxial compressive strength and the specimen diameter proposed by Hoek, a modified empirical relation is studied, and a calculation model of hole collapse pressure is established.

Compared with conventional well, the collapse pressure of slimhole is lower, and the slimhole is more stable.

The size effect of hole collapse pressure depends on the size effect of UCS. And UCS is a function of cohesion and the internal frictional angle. Usually cohesion decreases with the increase of specimen size. However, the change rule of the internal frictional angle may be complicated. When the internal frictional angle increases with size, the size effect of collapse pressure is weakened. And when the internal frictional angle decreases with size, the size effect of collapse pressure is strengthened.

Experimental method and logging method is proposed to evaluate the size effect of rock strength. With experimental method, the relation between the rock strength parameters and specimen size can be established. With the logging method, a fuzzy relation between fracture index and logging data is established to evaluate the fracture development level, and this still needs further research to study the relationship between rock strength and fracture index.

ACKNOWLEDGMENT

This research is supported by China National Offshore Oil Corporation. The authors wish to thank a number of people in CNOOC who contributed to this research.

REFERENCES


Jin Sun was born in Shandong province, China. He was born on February 17th, 1987. And he is currently a PH.D candidate at China University of Petroleum (Beijing), Department of Petroleum Engineering, Beijing, China. He is major in petroleum engineering which includes water flooding development, hydraulic fracturing and well stability. He entered China University of Petroleum (Huadong) in 2005, and got Bachelor degree in 2009. Then he was recommended for admission to graduate school after graduation in 2009. and in 2012, he got master degree in China University of Petroleum(Huadong). He has presented 2 paper: “Ground Anti-Collision monitoring and warning system for directional wells” and “Feasibility analysis on application of casing head vibration signal in wellbore anti-collision monitoring”. He has been involved in the area of anti-collision method during cluster well drilling, and currently he is major in water flooding development, hydraulic fracturing and well stability.

Jingen Deng was born in Jiangxi province, China. He was born in July, 1963, and he is a professor at China University of Petroleum (Beijing), Department of Petroleum Engineering, Beijing, China. He is major in petroleum engineering which include water flooding development, hydraulic fracturing, sand control technique, casing corrosion, well stability. He graduated from China University of Petroleum (Huadong) in 1983. And got his doctor’s degree in 2000 at China University of Petroleum (Beijing). He is currently the director of CNPC well stability laboratory, the director of Science and Technology Department of China University of Petroleum(Beijing).He has written over 60 technical publications on petroleum engineering. He has been involved in the well stability during PHD’s study, and now he is major in flooding development, hydraulic fracturing, sand control technique, casing corrosion, well stability. Prof. Deng was awarded as “Changjiang Scholar” by Ministry of Education in China, and He was selected as candidate of New Century Talents Project in China. He won the top 100 outstanding doctoral dissertation winner in 2003 and the second prize of National Scientific and Technological Progress Award.